9. Consistent framing in a strategic setting

- So far we assumed that output is affected by
  - Our own activities or choices
  - States
  - We considered single person decision problems

- This might not be appropriate to model some decisions/choices

- It might happen that output is affected not only by our *own* decisions but also by the choices of *others*
- *Strategic interaction* is present
  - If this is a first order effect it should be considered
  - Game theory is to be applied

Equilibrium Behavior

- Keeping things as simple as possible but considering strategic interaction output is now affected by
  - Our own activities or choices
  - The activities and choices of one more player
  - States
- Both players choose from a set of alternatives

$$a_1 \in A_1$$
$$a_2 \in A_2$$

- Criterion functions for both individuals are denoted:

$$\omega_1(a_1, a_2) \ , \ \omega_2(a_1, a_2)$$

- Uncertainty is typically present such that criterion functions are expected utility measures

Simultaneous Choice

- Assumptions:
- Both players know the sets of alternatives of both players, A_1 and A_2
- They know each others criterion functions $\omega_1(a_1, a_2), \omega_2(a_1, a_2)$

- The pair of choices is a (nash-) equlibrium if the following holds:

$$a_1^* \in \arg \max_{a_1 \in A_1} \omega_1(a_1, a_2^*)$$

$$a_2^* \in \arg \max_{a_2 \in A_2} \omega_2(a_1^*, a_2)$$

  – *Mutual best response of each player*

Sequential Choice

- Assuming sequential choice implies that
  - One player moves first
  - The second player observes this move
  - The second player moves based on this knowledge

- The game is solved by backwards induction
  - A reaction function is derived
    - Assume player 1 moves first and chooses $\widehat{a_1} \in A_1$
    - Player 2 chooses a_2 optimally given $\widehat{a_1}$
  - Optimization problem: $\max_{a_2 \epsilon A_2} \omega_2(\hat{a}_1, a_2)$
  - Repeating this for each possible choice of player 1 results in a reaction function:

$$\max_{a_1 \epsilon A_1} \omega_1(a_1, R(a_1))$$

- The optimal choice for player 1 is derived by maximizing the objective function w.r.t. a_1

$$\max_{a_1 \epsilon A_1} \omega_1(a_1, R(a_1))$$

- The equilibrium is characterized by $(a_1^*, R(a_1))$

- The criterion function value for each player *i* in equilibrium equals

$$\omega_i(a_1^*, R(a_1^*))$$

- Note: the optimal move of player 2 depends fully on the move of player 1 (first mover advantage)
- Essentially the third principle of consistent framing is applied:
  - The choice setting is transformed in a single person decision problem

Repeated choice

- What happens when a game is played repeatedly?
- Most simple setting: a game is played twice
- Before the second round starts both players know the moves and outcomes from the first round
- Backward induction is applied again
  - We start playing the second round
  - Second period (final round) equilibrium equals one shot game equilibrium
  - Given this knowledge first period equilibrium also equivalent to one shot game equilibrium
  - This holds for all finite games with known number of rounds to be played
- Another application of the third principle of consistent framing

Sharing a market

- Two firms compete for market share
- The market price depends on the total quantity offered
- Profit for each firm equals revenue less cost

$$\Pi_i(q_{1,}q_2) = \hat{P}(q_1 + q_2) * q_i - C(q_i;P)$$

- Note: the profit of each firm depends on the quantities chosen by both players
- The pair of output quantities that constitute a Nash equilibrium are defined as follows:

$$q_1^* \in \arg\max_{q_1 \geq 0} \pi_1(q_1, q_2^*)$$

$$q_2^* \in \arg\max_{q_2 \geq 0} \pi_2(q_1^*, q_2)$$

Racing to capture a Market

- Basic assumptions of the game:
  - First-mover advantage leads to capture of the whole market
  - The first to enter the market receives a "price" $\hat{P}$
  - Coming in second is of no value
  - Two competitors are present
  - One of them will secure the price
  - The probability of winning depends on the amount invested in R&D relative to the competitor
  - Investments are denoted z_1 and z_2
  - The probability to win the race is given by

$$p_i(z_1, z_2) = \frac{1 + z_i}{1 + z_1 + 1 + z_2} = \frac{1 + z_i}{2 + z_1 + z_2}$$

- Expected profit for each of the competitors equals

$$\Pi_i(z_1, z_2) = p_i(z_1, z_2) * \hat{P} - z_i = \frac{(1 + z_i)\hat{P}}{2 + z_1 + z_2} - z_i$$

- Both competitors simultaneously make their R&D decisions
- The following strategies constitute a Nash equilibrium

$$z_1^* \in \arg \max_{z_1 \geq 0} \pi_1(z_1, z_2^*)$$

$$z_2^* \in \arg \max_{z_2 \geq 0} \pi_2(z_1^*, z_2)$$

First order conditions for both competitors equal:

$$\frac{\partial \Pi_1(z_1, z_2^*)}{\partial z_1}|z_1 = z_1^* = \frac{(1 + z_2^*)\hat{P}}{(2 + z_1 + z_2^*)^2} - 1 = 0 \qquad \frac{\partial \Pi_2(z_1^*, z_2)}{\partial z_2}|z_2 = z_2^* = \frac{(1 + z_1^*)\hat{P}}{(2 + z_1^* + z_2)^2} - 1 = 0$$

- solving for z_i results in $z_i = \frac{1}{4}\hat{P} - 1$

Bidding for a prize

- The bidding game:
  - A customer asks for a customized product
  - Two potential bidders are present (firm 1 and firm 2)
  - If both suppliers hand in a bid the lower one wins
  - If both bidders submit the same bid the winner is randomly selected

  - The bids are submitted simultaneously
  - The bidders are risk neutral and face identical cost structures

$$\Delta = \alpha x + \beta y + \gamma z$$

  - x, y, and z are independent, identically distributed random variables with uniform densities between 0 and 1

- Expected incremental cost for each firm:

$$E[\Delta] = E[\alpha x + \beta y + \gamma z] = \alpha E[x] + \beta E[y] + \gamma E[z] = (\alpha + \beta + \gamma)/2$$

- What is the incremental gain of firm 1?

$$\Pi_1(b_1, b_2) = \begin{cases} 0 \;\; if \; b_1 > b_2 \\ b_1 - \Delta \; if \; b_1 < b_2 \\ .5(b_1 - \Delta) if \; b_1 = b_2 \end{cases}$$

- What about firm 2?
- The following bidding strategies constitute a Nash equilibrium:

$$b_1^* \in \arg\max_{b_1} E[\Pi_1(b_1, b_2^*)] \qquad b_2^* \in \arg\max_{b_2} E[\Pi_2(b_1^*, b_2)]$$

Equilibrium Bids

- Bidding the expected cost $E(\Delta) = b_1^* = b_2^*$ is equilibrium behavior for both firms
  - Given one firm bids expected cost the other cannot do better
  - If it bids higher, it looses for sure
  - If it bids lower, it makes an expected loss
  - Equilibrium expected profit is zero

Extension: Private information

- Both firms observe some information before bidding
  - Firm 1 observes x and y
  - Firm 2 observes x and z
  - Both firms know that the other one has received information

- Given the information both firms update expectations on cost

$$E[\Delta|x,y] = \alpha x + \beta y + \gamma E[z] = \alpha x + \beta y + \gamma/2$$

$$E[\Delta|x,z] = \alpha x + \beta E[y] + \gamma z = \alpha x + \beta/2 + \gamma z$$

- Two things change as compared to the previous story
  - Equilibrium will be described by bidding functions, depending on (x,y,z)
  - The bidding behavior of each firm conveys information

Implication of winning a bid

- Consider firm 1
  - It knows x and y
  - It submits a bid based on that knowledge
  - If it wins the bit, it learns something about z
    - Possibly z is higher than expected as firm 2 bids are increasing in z
  - Expected cost, given firm 1 won the bid, can be described as

$$E[\Delta|x, y, b_2 > b] = \alpha x + \beta y + \gamma E[z|b_2 > b]$$

- Based on that firm 1's expected profit equals

$$E[\Pi_1(b, b_2)|x, y] = 0 * prob\{b_2 < b\} + (b - E[\Delta|x, y, b_2 > b]) * prob\{b_2 > b\}$$
$$+.5(b - E[\Delta|x, y, b_2 = b]) * prob\{b_2 = b\}$$

- Strategies that constitute a (Bayesian) equilibrium are:

$$b_1^*(x,y) \in \arg\max_b E\left[\Pi_1\left(b, b_2^*(x,z)\right)\middle| x,y\right] \; \forall\, x,y \in [0,1]$$

$$b_2^*(x,z) \in \arg\max_b E[\Pi_2(b_1^*(x,y), b)|x,z] \; \forall\, x,z \in [0,1]$$

- Explicitly the following bidding functions form an equilibrium

$$b_1^*(x,y) = \alpha x + (\beta+\gamma)/2 + (\beta+\gamma)y/2$$

$$b_2^*(x,z) = \alpha x + (\beta+\gamma)/2 + (\beta+\gamma)z/2$$

The winner's curse

TABLE 10.1: Equilibrium Implications

bidding strategies
$b_1^*(x,y) = \alpha x + (\beta+\gamma)/2 + (\beta+\gamma)y/2$
$b_2^*(x,z) = \alpha x + (\beta+\gamma)/2 + (\beta+\gamma)z/2$
difference in bids
$b_1^*(x,y) - b_2^*(x,z) = \frac{\beta+\gamma}{2}(y-z)$
expected profit given information
$E[\Pi_1(b_1^*(x,y), b_2^*(x,z))\|x,y] = .5\beta(1-y)^2$
$E[\Pi_2(b_1^*(x,y), b_2^*(x,z))\|x,z] = .5\gamma(1-z)^2$
expected cost prior to bid
$E[\Delta\|x,y] = \alpha x + \beta y + \gamma/2$
$E[\Delta\|x,z] = \alpha x + \beta/2 + \gamma z$
revised expected cost if bid wins
$E[\Delta\|x,y,b_1 < b_2] = \alpha x + \beta y + \gamma(1+y)/2$
$E[\Delta\|x,z,b_2 < b_1] = \alpha x + \beta(1+z)/2 + \gamma z$
bias in initial cost estimate
$E[\Delta\|x,y,b_1 < b_2] - E[\Delta\|x,y] = \gamma y/2$
$E[\Delta\|x,z,b_2 < b_1] - E[\Delta\|x,z] = \beta z/2$
bid as expected cost plus markup
$b_1^*(x,y) = E[\Delta\|x,y] + \gamma y/2 + \beta(1-y)/2$
$b_2^*(x,z) = E[\Delta\|x,z] + \beta z/2 + \gamma(1-z)/2$

Haggling

- A single buyer and seller are present
- Cost of the seller is $\Delta=\alpha x$
- The value of the good to the buyer is V
- Social gain of a deal is $V-\Delta$ if $V>\Delta$

- How is the overall gain shared between the parties?
  - Nash bargaining solution: $(V-\Delta)/2$

- Private cost information
  - Only the seller knows the cost
  - Example: Δ is either 1 or 2 with equal probability, V=4
  - The buyer makes a "take it or leave it offer"
  - If the seller disagrees the game ends, otherwise the deal is made
  - Assume the odds are 0.8 for low cost and 0.2 for high cost instead

Internal Control

- Equilibrium analysis helps us to identify how choices are made in strategic settings
- It also shows that finer details of the game matter a great deal
- The accounting system
  - Is a system that receives input from many individuals
  - It's outcome is a result of various choices
  - The "accounting library" is subject to regulation
    - Regulation affects the finer details

      - Decision rights are limited
      - Redundancy is built in
      - Incentives are built in

  - This is what is often termed "***Internal Control***"