11. Large versus small decisions: long run

• Focus: decisions with long run consequences
  – Small and large decisions differ with regard to
    • Degree of variation on the status quo (do LLAs hold?)
    • Degree of interaction with other decisions
    • Degree of strategic interaction to be expected

  – We should expect that most of the long run decisions are large
    • Examples for small long term decisions, however, are available

• Appropriate approach for decisions with multi period consequences
  – With complete and perfect markets: present value
  – Without these: objectives are ambiguous
    • We rely on present value lacking an alternative
Present value

• The decision problem
  – An individual can choose an alternative \( a \) out of a set of alternatives \( A \)
  – The best choice maximizes the value of its criterion function
    \[
    \max_{a \in A} \omega(a)
    \]

• In the long run story
  – A choice \( a \) produces consequences (cash flows) over many periods
  – The time value of money plays a role
  – Each alternative gives rise to a different stream of cash flows
  – The criterion function is the present value of a cash flow stream
  – A discount rate needs to be specified
  – If uncertainty is present, expected present values are maximized
    \[
    a \in A \quad a = [x_0, x_1, \ldots, x_T]
    \]
• The criterion function then equals

\[ \omega(a) = PV(a) = \sum_{t=0}^{T} x_t (1 + r)^{-t} \]

• Example:
  – Two alternatives
    • Expand a business \((a_1)\)
    • Not expand \((a_2)\)
  – Expansion results in the following incremental cash flows

<table>
<thead>
<tr>
<th>( t = 0 )</th>
<th>( t = 1 )</th>
<th>( t = 2 )</th>
<th>( t = 3 )</th>
<th>( t = 4 )</th>
<th>( t = 5 )</th>
<th>( t = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-504</td>
<td>124</td>
<td>138</td>
<td>123</td>
<td>114</td>
<td>264</td>
<td>7</td>
</tr>
</tbody>
</table>
• PV(a_2)=0

• Assuming a discount rate of 12% we get

\[
PV(a_1) = -504(1.12)^0 + 124(1.12)^{-1} + 138(1.12)^{-2} + 123(1.12)^{-3} + 114(1.12)^{-4} \\
+ 264(1.12)^{-5} + 7(1.12)^{-6}
\]

\[= 30.07\]

• We obtain PV(a_1) > PV(a_2)
  – Expansion is preferred

• Two important questions arise:
  – Where do the cash flow estimates come from
  – Where does the discount rate come from
• Estimation of discount rate
  – Dependant on the project’s risk
  – Risk classes specified
  – A riskless project is discounted at the risk free rate
  – Risk class determined for the particular project or the whole firm
  – A firm’s cost of capital may determine the discount rate
  – Firms may use a fixed policy with respect to discount rates to be used

• Estimation of cash flows
  – Professional judgment is required here

• Alternative instruments might be used besides PV in order to make up for the estimation problems
Internal rate of return

• The discount rate that needs to be applied to get PV=0
  – Decision rule: if the IRR is higher than the discount rate, expansion should be chosen
  – Previous example:

\[
PV(a_1) = -504(1 + irr)^0 + 124(1 + irr)^{-1} + 138(1 + irr)^{-2} \\
+ 123(1 + irr)^{-3} + 114(1 + irr)^{-4} \\
+ 264(1 + irr)^{-5} + 7(1 + irr)^{-6} \\
= 0
\]

  – IRR = 14.1317%

• Problems:
  – Internal rate of return is not necessarily unique
  – Mutually exclusive projects cannot be compared based on IRR
Internal rate of return

• Example:
  • Assume an alternative generates the following cash flows
    \(a = [-100, 290, -208]\)

• Note that
  – There are two IRR
  – \(irr = 30\%\) and \(irr = 60\%\)
  – Lets assume the firm’s discount rate is 10%
    \[PV = -100 + 290(1 + r)^{-1} - 208(1 + r)^{-2}\]
    \(PV = -100 + 290(1.1)^{-1} - 208(1.1)^{-2} = -8.2545\)
  – Using PV the project is not favorable even though 10% < 30% < 60%
• Example:
• Assume two mutually exclusive projects are available
• The projects are characterized in the table below

<table>
<thead>
<tr>
<th>TABLE 12.2: Two Projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 0</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>choice $a_1$</td>
</tr>
<tr>
<td>choice $a_2$</td>
</tr>
</tbody>
</table>

• Note that the first project has the higher PV
• The second one has the higher irr
Payback

• Using the payback method we measure the length of time it needs for the cumulative cash flow of a project to be positive
  
  – This implies for a project \(a=[x_0,x_1,…,x_T]\) we require \(x_0+x_1+…..+x_{t_{PB}} \geq 0\)
  
  – Cash flows beyond \(t_{PB}\) are irrelevant
  
  – In its simplest form, no discounting is involved

• Payback is not equivalent to present value

• It measures risk as increasing with the time it takes cash flows to recover
  
  – Certainly a casual notion of risk

• However, a short (long) payback time can reasonably be interpreted as a good (bad) signal
Cash flow estimation

• In the long run decisions are likely to be large
  – LLAs need to be called into question
  – Competitive responses in product and factor markets are likely
  – Professional judgment is needed

• An illustration:
  – We start from the example in the previous chapter
  – Two products are produced in two production departments (subassembly and assembly)
  – Capacity constraints are present
    • Subassembly: \( q_1 + q_2 \leq 6,000 \)
    • Assembly: \( q_1 + 2q_2 \leq 10,000 \)
• The following LLAs were used to estimate cost:

**Subassembly:**
- \( DL_S = 10q_1 + 10q_2 \)
- \( DM_S = 110q_1 + 200q_2 \)

**Assembly:**
- \( DL_A = 40q_1 + 80q_2 \)
- \( DM_A = 12q_1 + 15q_2 \)

**Overhead LLA:**
- \( OV = 2,000,000 + 3.5(DL_S + DL_A) \)

• Selling prices are: \( \hat{P}_1 = 600 \quad \hat{P}_2 = 1,100 \)

• Contribution margins:

<table>
<thead>
<tr>
<th></th>
<th>( \hat{P}_1 )</th>
<th>( \hat{P}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>600</td>
<td>1,100</td>
</tr>
<tr>
<td>direct labor</td>
<td>50</td>
<td>90</td>
</tr>
<tr>
<td>direct material</td>
<td>122</td>
<td>215</td>
</tr>
<tr>
<td>variable overhead at 3.5(direct labor)</td>
<td>175</td>
<td>315</td>
</tr>
<tr>
<td>estimated marginal cost</td>
<td>347</td>
<td>620</td>
</tr>
<tr>
<td>contribution margin</td>
<td>253</td>
<td>480</td>
</tr>
</tbody>
</table>
Status quo

- Optimization problem:

\[
\Pi^* \equiv \max_{q_1, q_2 \geq 0} 253q_1 + 480q_2 - 2,000,000 \\
\text{s.t.} \quad q_1 + q_2 \leq 6,000 \\
\quad q_1 + 2q_2 \leq 10,000
\]

- With \( q_1^* = 2,000, \quad q_2^* = 4,000, \quad \Pi^* = 426,000 \)
Expansion project

- The project considered increases the capacity of each department by 1,500 units
- Capacity can be purchased at 300,000 for 5 years
- To estimate cash flows we need to foretell the use of capacity:

\[
\tilde{\Pi}^* = \max_{q_1, q_2 \geq 0} 253q_1 + 480q_2 - 2,000,000
\]

s. t. \quad q_1 + q_2 \leq 6,000 + 1,500

\quad q_1 + 2q_2 \leq 10,000 + 1,500

Solution: \quad q_1^* = 3,500, \quad q_2^* = 4,000, \quad \tilde{\Pi}^* = 805,500

- Assume the gain is all cash relevant
- An investment of 300,000 leads additional cash inflow of 379,500 for 5 years
  - So far we have assumed the decision is small
A large decision?

• If the decision is large other alterations have to be expected
  – Selling prices
    • Assume that the selling price for product 1 goes down by 1,5% if the output is expanded
  – LLAs
    • Capacity increase may alter the production process
    • It might be necessary to treat overhead of the departments separately

• \( OV_S = 1,000,000 + 0.4DMS \)
• \( OV_A = 1,200,000 + 3DL_A \)
• Cost and CM estimates become:

<table>
<thead>
<tr>
<th></th>
<th>591</th>
<th>1,100</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>direct labor in subassembly (DL₅)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>direct material in subassembly (DM₅)</td>
<td>110</td>
<td>200</td>
</tr>
<tr>
<td>subassembly variable overhead: .4DM₅</td>
<td>44</td>
<td>80</td>
</tr>
<tr>
<td>direct labor in assembly (DL₄)</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>direct material in assembly (DM₄)</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>assembly variable overhead: 3DL₄</td>
<td>120</td>
<td>240</td>
</tr>
<tr>
<td>estimated marginal cost</td>
<td>336</td>
<td>625</td>
</tr>
<tr>
<td>contribution margin</td>
<td>255</td>
<td>475</td>
</tr>
</tbody>
</table>

• Optimization program with revised terms

\[
\Pi^* \equiv \max_{q_1, q_2 \geq 0} 255q_1 + 475q_2 - 2,200,000
\]

\[
s.t. \quad q_1 + q_2 \leq 6,000 + 1,500
\]
\[
q_1 + 2q_2 \leq 10,000 + 1,500
\]

– Solutions are: \( q_1^* = 3,500, q_2^* = 4,000, \Pi^* = 592,500 \)
• The annual incremental gain from the expansion is now

\[ \pi^* - \Pi^* = 592,500 - 426,000 = 166,500 \]

• Further expansion costs:
  – Additional cost for training workers and altering the production facilities up to 90,000
    • Immediate cash outflow adds up to 390,000
    • We assume that additional working capital of 150,000 is needed at \( t=0 \) and returned at the end of the projects life
Effect on accounting income

• So far we used PV based on cash flows
• Accrual accounting is typically at odds with timing of cash flow consequences
• We need some (more) assumptions:
  – Most of the cash flow from operations will be recorded as income in the period received
  – Investment costs, however, will be allocated over the useful live of the project
  – The up-front alteration expenditure of 90,000 could be expensed immediately or allocated
    • We assume an immediate expense here
## TABLE 12.4: Incremental Accounting Income (000)

<table>
<thead>
<tr>
<th></th>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
<th>$t = 4$</th>
<th>$t = 5$</th>
<th>$t = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>cash from operations</td>
<td>0</td>
<td>167</td>
<td>167</td>
<td>167</td>
<td>167</td>
<td>167</td>
<td>0</td>
</tr>
<tr>
<td>accruals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>depreciation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>alteration</td>
<td>90</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>pretax income</td>
<td>-90</td>
<td>107</td>
<td>107</td>
<td>107</td>
<td>107</td>
<td>107</td>
<td>0</td>
</tr>
<tr>
<td>book tax at 40%</td>
<td>-36</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>0</td>
</tr>
<tr>
<td>income</td>
<td>-54</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>0</td>
</tr>
</tbody>
</table>