Stock price versus earnings based compensation in a dynamic agency setting*

Barbara Schöndube-Pirchegger
Otto-von-Guericke University Magdeburg
Universitätsplatz 2, 39106 Magdeburg
e-mail: barbara.schoendube@ovgu.de

Jens Robert Schöndube
Eberhard Karls University Tübingen
Nauklerstr. 47, 72074 Tübingen
e-mail: jensrobert.schoendube@uni-tuebingen.de

March 18, 2013

Abstract

We compare performance measurement based on earnings versus stock prices in a dynamic stewardship setting. Measures available for contracting are earnings and some macroeconomic measure. While earnings are informative about the manager’s effort and the firm’s terminal value, the macroeconomic measure is informative about terminal value only. Besides, the firm’s stock price can be contracted upon. We show that with full commitment the principal never uses the macroeconomic measure due to its lack of (effort-) informativeness. As the stock price inevitably includes this measure, using purely earnings based contracts outperforms using stock prices. Under limited commitment, however, we find that contracting on stock prices may be optimal. Intuitively, the principal sets lower incentives to overcome inefficient aggregation of information in stock prices. Thus using stock acts as a commitment to low incentives. We derive conditions for the optimality of stock price measurement in two scenarios: autocorrelated noise and earnings management.

---

*We gratefully acknowledge helpful comments by Carolin Mauch, Konrad Lang and participants of the Accounting Seminar in Tuebingen 2011.
Stock price versus earnings based compensation in a dynamic agency setting

Abstract: We compare performance measurement based on earnings versus stock prices in a dynamic stewardship setting. Measures available for contracting are earnings and some macroeconomic measure. While earnings are informative about the manager’s effort and the firm’s terminal value, the macroeconomic measure is informative about terminal value only. Besides, the firm’s stock price can be contracted upon.

We show that with full commitment the principal never uses the macroeconomic measure due to its lack of (effort-) informativeness. As the stock price inevitably includes this measure, using purely earnings based contracts outperforms using stock prices. Under limited commitment, however, we find that contracting on stock prices may be optimal. Intuitively, the principal sets lower incentives to overcome inefficient aggregation of information in stock prices. Thus using stock acts as a commitment to low incentives. We derive conditions for the optimality of stock price measurement in two scenarios: autocorrelated noise and earnings management.

Keywords: Stock Prices, Limited Commitment, Correlated Noise, Earnings Management, Compensation
1 Introduction

An extensive literature in accounting and finance discusses whether management compensation contracts should be based on the firm’s stock price, on accounting earnings, or on both.¹

Intuitively the stock price appears to be the most natural candidate for contracting. It aggregates backward as well as forward looking information and is therefore less “myopic” than earnings. It is presumed to be harder to manipulate for the management than earnings are. Last but not least, maximizing stock price is what shareholders are primarily interested in.

Contributions from the analytical agency literature attenuated this perception of stock prices as an optimal measure with respect to at least two aspects. First, stock prices are likely to be very noisy measures of managerial performance. They are affected by macroeconomic shocks beyond the management’s control. If managers are risk averse, using such a measure to create incentives comes along with high risk premiums to be paid. Second, and probably even more important, information is aggregated within the stock price in order to provide a best estimate of firm value. As opposed to that an optimal compensation contract aggregates information that is informative about the manager’s effort. Both paradigms typically do not coincide. Accordingly, the pieces of information included in the stock price as well as the weights used for aggregation in most cases differ from what would be optimal for contracting.

In this paper we address the question of stock price versus earnings in a dynamic context. To carve out our point we create, as a starting point, a setting in which it is suboptimal to include the stock price at all in the full commitment incentive contract. Rather, using earnings only turns out to be optimal. Relaxing the full commitment assumption and considering renegotiation-proof contracts instead, however, we are able to show that in some settings stock price turns out to be preferable to earnings. Specifically, limited commitment may result in too high powered second period incentives when earnings are used for performance measurement. Using market prices instead is more costly, leading to more moderate incentives. While this is detrimental in the first period, it is favorable in the second and may be beneficial overall. Thus, we provide an additional argument in favor of using stock prices. This argument is especially valid in situations where limited rather than full commitment appears to be a reasonable assumption.

We use a two period LEN-type agency model. The agent performs a single productive effort in each period. Additionally he may perform an earnings management activity in the first period that reverses in the second. The principal wants to maximize the firm’s terminal payoff. While this payoff is assumed to be non-contractible, different performance measures are available for

¹See e.g. Lambert (1993), Bushman and Indjejikian (1993), Kim and Suh (1993), Sloan (1993).
contracting in each period. First, the principal can contract upon accounting earnings and some macroeconomic measure publicly available from outside the firm. Second, she can use the market price that aggregates the two former measures. While earnings are informative (in a Holmström (1979) sense) about the agent’s effort, the macro measure is assumed not to be. We distinguish two settings. In the first setting we assume that each performance measure is correlated over periods but no earnings management is possible. In the second we allow for earnings management but neglect autocorrelation. Doing so we make sure in both settings - in different fashion- that the periods are interdependent.

Our model setup builds on findings from at least three different streams of literature. One of them is the literature concerned with limited commitment. If long term commitment is infeasible, the equilibrium outcome is determined by sequentially rational contracting decisions. Ex post efficient contracts, however, may well be inefficient from an ex ante perspective. This inefficiency results in a loss in welfare from limited commitment that can be avoided in special cases only (see Fudenberg et al. (1990)). In a two-period LEN-setting Indjejikian and Nanda (1999) and Christensen et al. (2003, 2005) show that limited commitment generally creates a welfare loss if performance measures are inter-temporally correlated. Their result naturally extends to our paper.

Moreover, in a recent paper Schöndube-Pirchegger and Schöndube (2012) show that delegation of decision rights may serve as an commitment to higher powered incentives in an agency with limited commitment. As opposed to that we show in this paper that market price based incentive contracts may be used as a commitment to low incentives.

Another stream of literature investigates the optimal aggregation of performance measures in incentive contracts. Gjesdal (1981) shows that aggregation for stewardship purposes typically differs from aggregation for valuation. Building on that Feltham and Xie (1994) demonstrate that contracting on a market price that aggregates accounting information only, is likely to be inferior to a contract that uses the very same accounting measures directly. The inefficiency of stock prices following from suboptimal information aggregation is further demonstrated in Paul (1992).

Finally, our paper ties in with the literature on earnings management. Similar to Feltham and Xie (1994) we model earnings management as an effort affecting the performance measure but not the firm’s terminal payoff. As in Fischer and Verrecchia (2002), earnings measurement arises even though it is perfectly anticipated by the market and corrected for in a rational expectations equilibrium.
The next section introduces the basic model. Section 3 characterizes the benchmark solution with full commitment. Section 4.1 derives conditions for the optimality of market price measurement with autocorrelated noise, and section 4.2 does so when the agent is able to conduct earnings management. Section 5 concludes.

2 The model

We consider a two-period LEN-model of repeated moral hazard. At the beginning of the first period the principal hires an agent to perform a certain task \( a_t \) in each period \( t \). The firm’s terminal value is given by

\[
x = a_1 + a_2 + \varepsilon_x.
\]

\( \varepsilon_x \) is a normally distributed noise term with zero mean and variance \( \sigma_x^2 \). While \( x \) is non-contractible information, two verifiable performance measures, earnings and a macroeconomic shock term, are available in each period \( t = 1, 2 \):

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earnings</strong></td>
<td><strong>Earnings</strong></td>
</tr>
<tr>
<td>( y_{11} = g_1a_1 + \delta e + \varepsilon_{11} )</td>
<td>( y_{21} = g_2a_2 - \delta e + \varepsilon_{21} )</td>
</tr>
<tr>
<td><strong>Macroeconomic shock</strong></td>
<td><strong>Macroeconomic shock</strong></td>
</tr>
<tr>
<td>( y_{12} = \varepsilon_{12} )</td>
<td>( y_{22} = \varepsilon_{22} )</td>
</tr>
</tbody>
</table>

We assume that the manager’s action \( a_t \) affects earnings but not the shock in both periods. \( g_t \) are positive parameters determining to what extent the manager’s effort affects earnings. Furthermore, earnings are potentially affected by an earnings management activity \( e \). Earnings management allows the manager to increase one period’s earnings at the cost of the other period’s earnings. If he exerts positive earnings management effort, \( e > 0 \), he moves earnings from the second into the first period. With negative earnings management effort he postpones earnings from the first into the second period. Overall earnings remain unaffected no matter what the manager does.\(^2\) Assuming a quadratic disutility of \( e^2/2 \) from conducting earnings management both types of earnings management effort are identically costly to the agent.

The above setup allows us to introduce two types of inter-temporal dependencies. One is to assume that earnings as well as shocks are autocorrelated. The other one to allow for earnings management. Both alternatives are considered separately later on. To disentangle the two we

\(^2\) Using the above assumptions we model accounting earnings management rather than real earnings management in an Ewert and Wagenhofer (2005) sense. The agent manages accounting earnings but not real transactions. Furthermore, our presumed accounting system is one that requires earnings to equal cash flows over the total life of the firm (clean surplus accounting).
assume \( \text{corr}(y_{11}, y_{21}) = \lambda \) and \( \text{corr}(y_{12}, y_{22}) = \mu \) with \( \lambda, \mu \neq 0 \) in the first setting but neglect earnings management: \( \delta = 0 \) and \( g_1 = g_2 = 1 \). In the second setting we allow for earnings management, that is \( \delta > 0 \) and \( g_1 \neq g_2 \), but set \( \lambda = \mu = 0 \). For convenience we sum up the parameter specifications in Table 1.

<table>
<thead>
<tr>
<th>setting</th>
<th>specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>autocorrelated noise</td>
<td>( \delta = 0, g_1 = g_2 = 1 )</td>
</tr>
<tr>
<td>earnings management</td>
<td>( \lambda = \mu = 0, g_1 \neq g_2 )</td>
</tr>
</tbody>
</table>

Table 1: Two different settings to be analyzed: autocorrelated noise and earnings management

The agent’s personal cost from performing productive effort \( a_t \) is assumed to be \( a_t^2 / 2 \). We denote the vector of the agent’s actions as \( \varepsilon = (a_1, e, a_2)' \) and its cost as \( C(\varepsilon) = \varepsilon' \varepsilon / 2 \) in what follows.\(^4\)

Like \( \varepsilon \) the random variables \( (\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{22}) \) are normally distributed with means of zero and variances of \( \sigma^2 \). For simplicity we assume that earnings and macroeconomic shock terms are not stochastically correlated.\(^5\) Intuitively this might well be the case if, e.g., earnings are entirely backward looking while the macroeconomic measure reflects future expectations. Finally, we assume that the correlation between first-period performance measures \( (y_{11}, y_{12}) \) and \( x \) is \( v_1 \) and between \( (y_{21}, y_{22}) \) and \( x \) it is \( v_2 \) such that the following covariance matrix for \( \varepsilon = (\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{22}, \varepsilon_x) \) results:

\[
\mathbf{\Sigma}_{(\varepsilon_{11, \varepsilon_{12, \varepsilon_{21, \varepsilon_{22, \varepsilon_x}}})} = \begin{pmatrix}
\sigma^2 & 0 & \lambda \sigma^2 & 0 & v_1 \sigma \varepsilon_x \\
0 & \sigma^2 & 0 & \mu \sigma^2 & v_1 \sigma \varepsilon_x \\
\lambda \sigma^2 & 0 & \sigma^2 & 0 & v_2 \sigma \varepsilon_x \\
0 & \mu \sigma^2 & 0 & \sigma^2 & v_2 \sigma \varepsilon_x \\
v_1 \sigma \varepsilon_x & v_1 \sigma \varepsilon_x & v_2 \sigma \varepsilon_x & v_2 \sigma \varepsilon_x & \sigma^2_x
\end{pmatrix}
\]

We require \( \mathbf{\Sigma} \) to be a positive definite matrix which rules out, e.g., extreme cases like \( \lambda \) or \( \mu \) equal to \( -1 \) or \( \sigma = 0 \). Let \( \mathbf{y}_1 = (y_{11}, y_{12})' \), \( \mathbf{y}_2 = (y_{21}, y_{22})' \) and \( \mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2)' \). We can define the

---

\(^{3}\) \( g_1 \neq g_2 \) is necessary for earnings management to occur in equilibrium. To ease the analysis we set \( g_1 = g_2 = 1 \) in the autocorrelated noise setting.

\(^{4}\) \( \times \) denotes the transpose of a matrix or a vector.

\(^{5}\) This assumption is not necessary to derive our results but reduces complexity considerably.
following sub-matrices of $\Sigma$ for partitions of $\epsilon$:

$$
\Sigma_y = \begin{pmatrix}
\sigma^2 & 0 & \lambda \sigma^2 & 0 \\
0 & \sigma^2 & 0 & \mu \sigma^2 \\
\lambda \sigma^2 & 0 & \sigma^2 & 0 \\
0 & \mu \sigma^2 & 0 & \sigma^2
\end{pmatrix},
\Sigma_{y_1} = \begin{pmatrix}
\sigma^2 & 0 \\
0 & \sigma^2
\end{pmatrix},
\Sigma_{y_2,y_1} = \begin{pmatrix}
\lambda \sigma^2 & 0 \\
0 & \mu \sigma^2
\end{pmatrix}.
$$

The agent is strictly risk averse with utility $U^A = -\exp(-r(S - C(\epsilon)))$. Here $S$ denotes the agent’s compensation and $r > 0$ is the agent’s risk aversion coefficient. We restrict attention to two-period incentive contracts that are linear in the performance measures. This assumption combined with exponential utility and normality leads to the well known LEN-specification. The agent’s preferences can be represented by

$$CE(S, \epsilon) = E(S) - C(\epsilon) - \frac{r}{2}Var(S).$$

The certainty equivalent of the agent’s reservation utility is normalized to zero.

The principal is risk neutral. She chooses performance measures and optimal contracting coefficients to maximize her net outcome $U = E(x - S)$.

With respect to the agent’s compensation contract, we distinguish two performance measurement systems: Separate measurement (SM) and aggregate or market price measurement (MP). Both, the individual measures and the market prices are publicly available information. With separate measurement the linear contract becomes

$$S^{SM} = f + s_{11}y_{11} + s_{12}y_{12} + s_{21}y_{21} + s_{22}y_{22}.$$  

Here $f$ denotes a fixed payment and $s_{ij}$ is the incentive weight for performance measure $y_{ij}$. When market prices are used for performance measurement the agent’s compensation is defined as

$$S^{MP} = f + s_1P_1 + s_2P_2.$$  

$P_t$ denotes the market price of the firm at time $t$ and $s_1$ and $s_2$ are incentive weights. We assume a competitive market with risk neutral investors. All information available in the market, earnings and the macroeconomic measure in our case, is shared by everyone. In such a setting the market price $P_t$ is defined as the expected terminal firm value, conditional on the information available
at $t$ and conditional on rational conjectures with respect to unobservable actions. Specifically,

$$
P_1 = E(x|y_1, \hat{z}),$$

$$P_2 = E(x|y, \hat{z})$$

where $\hat{z}$ denotes the market’s conjectures about the agent’s effort $z$, which are consistent with actual behavior in equilibrium. Given that $e$ is normally distributed, market prices given in (1) can be written as

$$
P_1 = K_1 + \beta_1 y_1$$

$$P_2 = K_2 + \beta_2 y,$$

where $\beta_1 = \Sigma_{y_1,z} \Sigma_{y_1}^{-1}$, $K_1 = E(x|\hat{z}) - \Sigma_{y_1,z} \Sigma_{y_1}^{-1} E(y_1|\hat{z})$, $\beta_2 = \Sigma_{y,z} \Sigma_{y}^{-1}$, and $K_2 = E(x|\hat{z}) - \Sigma_{y,z} \Sigma_{y}^{-1} E(y|\hat{z})$, or equivalently,

$$
P_1 = K_1 + \beta (y_{11} + y_{12}),$$

$$P_2 = K_2 + (\beta_{11} y_{11} + \beta_{12} y_{12}) + (\beta_{21} y_{21} + \beta_{22} y_{22}),$$

with $\beta = \frac{\sigma_x}{\sigma} v_1, \beta_{11} = \frac{\sigma_x}{\sigma} \frac{\lambda \nu_1 - v_1}{(\lambda - 1)(\lambda + 1)}, \beta_{12} = \frac{\sigma_x}{\sigma} \frac{\nu_1 - v_1}{(\mu - 1)(\mu + 1)}, \beta_{21} = \frac{\sigma_x}{\sigma} \frac{\lambda \nu_1 - v_2}{(\lambda - 1)(\lambda + 1)},$ and $\beta_{22} = \frac{\sigma_x}{\sigma} \frac{\nu_1 - v_2}{(\mu - 1)(\mu + 1)}.$

To exclude trivial cases we assume parameter settings where $\beta$ and the $\beta_{1,2}$s are unequal to zero throughout the whole analysis.

As both, earnings and the macroeconomic shock measure, are informative about the firm’s terminal payoff the market price of period $t$ depends on both measures as far as observed until the end of period $t$. In particular the first-period market price weights earnings and shock equally. Whether this holds for the second-period market price as well depends on the setting. In the earnings management setting $\lambda = \mu = 0$, implying that second-period weights are equal as well. With autocorrelation, however, different weights may arise. The point is that $y_{11}$ and $y_{12}$ have been observed at the end of the first-period. With autocorrelation (given by $\lambda$ and $\mu$, respectively) the market learns something about second-period variables $y_{21}$ and $y_{22}$ from this observation but not necessarily the same. Whenever $\lambda \neq \mu$, i.e., the correlation differs for earnings and the shock term, second-period market price weights both measures differently.

### 3 Benchmark: Full commitment solutions

In this section we characterize the full commitment setting. As this setting serves as a benchmark only we keep it short and relegate the presentation of most of the analysis to the appendix. The

---

\[6\] See, e.g., Paul (1992) and Feltham and Xie (1994).
The risk neutral principal maximizes her net return subject to two conditions. The individual rationality constraint (IR) is binding at the optimum and ensures that the agent accepts the contract. Further, the agent chooses his actions in order to maximize personal welfare. This is reflected in the incentive compatibility constraints (IC).

The specifics of the problem differ depending on the scenario considered. As stated above we distinguish two types of inter-temporal dependencies, an autocorrelated noise setting and an earnings management setting. Moreover, two measurement systems, (SM) and (MP), are considered. Accordingly, four different maximization problems need to be solved.

The following lemma points towards an important property of both types of optimal (SM)-contracts that further extends to the limited commitment setting.

**Lemma 1** Under (SM) the macroeconomic shock measures are not elements of the optimal managerial incentive contract, i.e. \( s_{12} = s_{22} = 0 \). This result holds with full as well as limited commitment.

**Proof.** Neither \( y_{12} \) nor \( y_{22} \) are informative about \( \kappa \). \( E \left( y_{12} | \kappa, y_{11}, y_2 \right) = \mu_{y_{22}} \) and \( E \left( y_{22} | \kappa, y_1, y_{21} \right) = \mu_{y_{12}} \) do not depend on \( \kappa \) such that the conditional normal densities \( f_{y_{12}} \left( y_{12} | \kappa, y_{11}, y_2 \right) \) and \( f_{y_{22}} \left( y_{22} | \kappa, y_1, y_{21} \right) \) do not depend on \( \kappa \), either. A non-informative measure will never be used in an optimal incentive contract, no matter whether it is used ex ante or ex post efficient. It cannot improve the trade-off between risk and incentives. ■

Given the structure of our model, the macroeconomic shocks are not informative as they neither influence the agent’s action choices nor are they correlated to earnings. Thus, observing this measure does not add anything to the information about actions already contained in earnings.

Below we present optimal incentive rates for all four settings. Details are given in appendix A. Given the result from lemma 1, solutions for \( s_{11} \) and \( s_{21} \) only are shown with separate measurement.

With market price measurement we skip to present \( s_1 \) and \( s_2 \) and rather refer to what we call effective incentive rates, \( \bar{s}_1 \) and \( \bar{s}_2 \). The effective incentive rates show the full effect of effort...
(via earnings) on the agent’s compensation in the (MP) setting as do $s_{11}$ and $s_{21}$ in the (SM) setting.

$$\overline{s}_1 = s_1 \beta + s_2 \beta_{11} \quad (3)$$

$$\overline{s}_2 = \beta_{21} s_2.$$

In particular we observe from the effective first-period incentive rate, $\overline{s}_1$, that first-period effort via first-period earnings affects the market prices of both periods. $\beta, \beta_{11}$ and $\beta_{21}$ represent the weights attached to earnings in the pricing process. Representing the full effect of effort on compensation effective rates allow to properly compare the (MP) to the (SM) setting.

<table>
<thead>
<tr>
<th>Autocorrelated noise setting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SM:</strong></td>
</tr>
<tr>
<td>$s_{11}^* = \frac{1}{1+r\sigma^2(\lambda+1)}$</td>
</tr>
<tr>
<td>$s_{21}^* = \frac{1}{1+r\sigma^2(\lambda+1)}$</td>
</tr>
<tr>
<td><strong>MP:</strong></td>
</tr>
<tr>
<td>$\overline{s}<em>1 = \frac{r\sigma^2(\beta</em>{12}\beta_{21}+2\beta\beta_{21}(\beta_{11}-\beta_{12})+2\beta_{22}\beta_{21}\mu+\beta_{21}^2(\lambda-1)-\beta_{11}\beta_{12}+2\beta_{11}^2+\beta_{12}^2-\beta_{11}-\beta_{12}}{n}$</td>
</tr>
<tr>
<td>$\overline{s}<em>2 = \beta</em>{21} \frac{r\sigma^2(\beta_{21}(\lambda-1)+\beta_{12}+\beta_{22}\mu-2\beta_{21})}{n}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Earnings management setting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SM:</strong></td>
</tr>
<tr>
<td>$s_{11}^* = \frac{\delta^2 g_2 + g_1 (g_2^2 + \delta^2 + r\sigma^2)}{g_2^2 g_1^2 + (g_2^2 + g_1^2) (\delta^2 + r\sigma^2) + 2r\sigma^2 + r^2 \sigma^4}$</td>
</tr>
<tr>
<td>$s_{21}^* = \frac{\delta^2 g_1 + g_2 (g_2^2 + \delta^2 + r\sigma^2)}{g_2^2 g_1^2 + (g_2^2 + g_1^2) (\delta^2 + r\sigma^2) + 2r\sigma^2 + r^2 \sigma^4}$</td>
</tr>
<tr>
<td><strong>MP:</strong></td>
</tr>
<tr>
<td>$\overline{s}_1 = \frac{g_1 (g_2^2 + \delta^2 + 2r\sigma^2) + g_2 \delta^2}{(\delta^2 + 2r\sigma^2) (g_2^2 + g_1^2) + g_2^2 g_1^2 + 4r\sigma^2 (\delta^2 + r\sigma^2)}$</td>
</tr>
<tr>
<td>$\overline{s}_2 = \frac{g_1 (g_1^2 + \delta^2 + 2r\sigma^2) + g_2 \delta^2}{(\delta^2 + 2r\sigma^2) (g_2^2 + g_1^2) + g_2^2 g_1^2 + 4r\sigma^2 (\delta^2 + r\sigma^2)}$</td>
</tr>
</tbody>
</table>

Table 2: Full commitment incentive weights

The incentive rates will become useful later on. For now they help to derive the essential result from this section.

**Lemma 2** Under full commitment separate measurement (SM) strictly dominates market price measurement (MP).

$$n = \beta [-2r^2\sigma^4 \{(\beta_{11} - \beta_{12}) (\lambda \beta_{21} - \mu \beta_{22}) + \beta_{21}^2 + \beta_{22}^2 - \beta_{11} \beta_{12} - \beta_{21} \beta_{22} \lambda \mu \}
+ 2r \sigma (\beta_{11} \beta_{12} + \beta_{22} \mu (\beta_{11} - \beta_{12})) - r \sigma^2 (\beta_{11}^2 + \beta_{12}^2 + 2\beta_{21}^2 + \beta_{22}^2)
+ r^2 \sigma^4 (\beta_{12} (\lambda^2 - 1) + \beta_{22} \mu^2 - \beta_{11}^2) - \beta_{21}^2]$$
Proof. See appendix B. □

Earnings and macroeconomic shocks are informative about the firm’s final payoff. Accordingly, the market price, and in turn a market price based contract, relies on both measures. The macroeconomic shocks, however, are not informative about the manager’s actions. It follows that under (MP) the optimal contract makes inefficiently use of these measures. Doing so the optimal trade-off between insurance and incentives cannot be induced in a full commitment setting. With this result in place we continue comparing both measurement systems again in a limited commitment setting.

4 The use of market price measurement as a commitment device

4.1 Limited commitment and renegotiation-proofness in a two-period LEN-setting

In this section we assume that contracting parties are unable to commit to a long-term (two-period) incentive contract. In other words they cannot preclude ex ante to renegotiate an inefficient contract $S$ ex post. $S$ may become inefficient for two reasons: Either a different ex post trade-off between risk and incentives arises after some uncertainty has been resolved (autocorrelated noise setting), or long-term consequences of first-period actions become irrelevant from a second-period perspective (earnings management setting).

In both cases the principal is free to offer a revised contract $S^R$ to the agent at the end of the first period. At that stage of the game $y_{11}$ and $y_{12}$ have been observed and the agent has performed $a_1$ and possibly $e$. The agent will accept the new contract if he is at least indifferent between $S$ and $S^R$. Literally, this kind of renegotiation procedure can take place at any time during the two-period relation. The end of the first-period, however, appears to be most self-evident for the contractual relationship considered here. Typically compensation committees meet annually and adapt managerial compensation at the end of a period.\footnote{See Christensen et al. (2003).}

From the literature we know that under complete contracts it is not necessary to analyze the renegotiation procedure explicitly. Without loss of generality one can concentrate on initial contracts that are robust against renegotiation (renegotiation-proof).\footnote{See Fudenberg and Tirole (1990) and Christensen et al. (2003).} Christensen et al. (2003) prove the renegotiation-proofness-principle for a two-period LEN-model and show that an initial contract is renegotiation-proof if and only if second-period incentive weights are chosen sequentially optimal. In other words the optimization problem to be considered is identical to the
one from the commitment setting except that renegotiation proof second-period incentive rates apply. In what follows we use this approach. With \( s_{12} = s_{22} = 0 \) from lemma 1, a contract under (SM) is renegotiation-proof in our setting iff \( s_{21} \) is set sequentially optimal. Under (MP) a contract is renegotiation-proof iff \( s_2 \) (or equivalently, \( \bar{s}_2 \)) is set sequentially optimal.

### 4.2 Market price measurement as a commitment to low incentives in an autocorrelated noise setting

In the autocorrelated noise setting the agent is not able to conduct earnings management and earnings are given by

\[
\begin{align*}
y_{11} &= a_1 + \varepsilon_{11} \\
y_{21} &= a_2 + \varepsilon_{21}.
\end{align*}
\]

This implies that any frictions between ex ante and ex post efficient second-period incentive rates can arise from autocorrelation only. In a first step we determine the sequentially optimal second-period incentive rate for each performance measurement system. We denote these values with superscript "\( R \)" for renegotiation-proofness.

**Lemma 3**

a) The renegotiation-proof second-period earnings weight under (SM) is given by

\[
s_{21}^R = \frac{1}{1 + r \sigma^2 (1 - \lambda^2)}.\]

b) The renegotiation-proof effective second-period weight under (MP) is given by

\[
\bar{s}_2^R = \frac{\beta_{21}^2}{\beta_{21}^2 (1 + r \sigma^2 (1 - \lambda^2)) + \beta_{22}^2 r \sigma^2 (1 - \mu^2)}.\]

**Proof.** See appendix B. ■

Consider the second-period renegotiation-proof earnings weight under separate measurement, \( s_{21}^R \). \( s_{21}^R \) trades-off second-period effort and second-period compensation risk conditional on \( y_{11} \).

In particular \( s_{21}^R \) accounts for the posterior variance \( Var (y_{21} | y_{11}) = \sigma^2 (1 - \lambda^2) \). The second period macroeconomic measure will not be contracted upon under (SM).

Under market price contracting, the principal inevitably contracts on both, earnings and the macroeconomic shock, if she wants to induce effort at all. Again the posterior variance of the agent’s compensation matters but now autocorrelated macro measures are added. This is captured by the term \( \beta_{22}^2 \sigma^2 (1 - \mu^2) \).
In the following lemma we compare second-period ex ante and ex post efficient earnings weights under (SM). We also show how second-period renegotiation-proof (effective) incentives differ across both measurement systems:

**Lemma 4**

a) Under separate measurement, the following relations apply: $s_{21}^s \geq s_{21}^R$ iff $\lambda \leq 0 \iff a_{21}^{SM,*} \geq a_{21}^{SM,R}$ iff $\lambda \leq 0$. b) $s_{21}^R < s_{21}^s$.

Whether second-period renegotiation-proof incentives are too high or too low (or equal) compared to the ex ante efficient ones under (SM) depends on the sign of earnings correlation $\lambda$.\(^{10}\) In addition, part b) of the lemma states that ex post efficient second-period (effective) incentives under market pricing are always lower than under separate measurement. The latter results from the fact that under market price measurement macro shocks will become element of the second-period incentive contract. Similar to the full commitment setting, contracting on the macroeconomic measure is inefficient as it is not informative about second-period effort. In order to reduce the costs related to this inefficiency, sequentially optimal second-period effort incentives under (MP) are lower than under (SM) where only earnings will be contracted upon.

The important point here is that under limited commitment the ex post inefficiency due to aggregate measurement under (MP) may become efficient from an ex ante perspective. To see this, consider a situation where the principal induces too high second-period effort from the ex ante point of view under separate measurement. In this case she would be better off if she could credibly commit to a lower second-period incentive rate. Indeed, by choosing the market price measurement system, the principal implicitly commits to such lower second-period incentives. The positive effect of aggregation on second-period incentives potentially overcompensates the negative effect of using a non informative measure in both periods. A necessary condition for this effect to arise is provided in the following proposition:

**Proposition 1** A necessary condition for market price performance measurement to be optimal under limited commitment is: $\lambda > 0$.

**Proof.** For $\lambda = 0$, $s_{21}^s = s_{21}^R$ under (SM) and the full commitment solution will be induced under renegotiation-proofness. Under full commitment, however, as has been shown in lemma 2, separate measurement strictly dominates market price measurement. If $\lambda < 0$ under separate measurement the agent is induced to perform too low second-period effort with renegotiation-proofness. Under market price measurement induced effort $a_2$ is even lower (lemma 4). As

\(^{10}\)See also Indjejikian and Nanda (1999), Christensen et al. (2005), and Schöndube-Pirchegger and Schöndube (2012) for the impact of correlation in a dynamic agency.
objective functions in our LEN-setting are strictly concave, the stronger the deviation from the optimum the lower the corresponding objective function values. Hence, \( U_{SM,R} \geq U_{MP,R} \) for \( \lambda \leq 0 \). For \( \lambda > 0 \) \( a_{2}^{SM,R} \) is too high compared to \( a_{2}^{SM,*} \). Now, market pricing may outperform separate measurement as it potentially relaxes the principal’s renegotiation-proof constraint. Hence, if \( U_{MP,R} > U_{SM,R} \) then \( \lambda > 0 \). □

If earnings are positively correlated, \( \lambda > 0 \), limited commitment forces the principal to induce too high second-period effort under separate measurement. To see this, consider the full-commitment solution first: If \( \lambda \) is positive, intertemporal persistence effects of earnings increase compensation risk and in turn reduce effort incentives in both periods. With limited commitment, however, the principal has to set renegotiation-proof second-period incentives. This includes to choose the second-period incentive rate as if the first-period effort was done and first-period earnings have been already observed. For \( \lambda > 0 \) first-period earnings are informative about second-period earnings which shows up in the posterior variance, \( \sigma^2 (1 - \lambda^2) < \sigma^2 \). As ex post efficient incentives are based on the posterior variance \( s_{21}^* < s_{21}^{R} \) for \( \lambda > 0 \). In such a situation, aggregation of both measures under market price contracting leads to lower second-period incentives. As a result, market price measurement may relax the renegotiation-proof constraint relative to separate measurement if \( \lambda > 0 \). Installing a (MP)-system the principal implicitly commits to (relatively) low second-period incentives. This commitment is beneficial if it outweighs the cost of using a non-informative measure for contracting in both periods. Due to the large set of parameters \( (r, \sigma, \sigma_x, \lambda, \mu, v_1, v_2) \) included in this model it is difficult to provide robust sufficient conditions for the optimality of market price measurement that cover a wide range of parameter constellations. We rather provide a numerical example to add intuition to the above proposition.

**Example 1:** Parameters of the example: \( \sigma = 1.6, \sigma_x = 1, \lambda = 0.76, \mu = -0.9, v_1 = 0.2, v_2 = 0.2, r = 1 \).

<table>
<thead>
<tr>
<th>Separate Measurement</th>
<th>Market Price Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC</td>
<td>FC</td>
</tr>
<tr>
<td>( s_{11}^* = 0.182 )</td>
<td>( s_1^* = 0.162 )</td>
</tr>
<tr>
<td>( s_{21}^* = 0.182 )</td>
<td>( s_2^* = 0.0224 )</td>
</tr>
<tr>
<td>( a_{1}^{SM,*} = 0.182 )</td>
<td>( a_1^{MP,*} = 0.162 )</td>
</tr>
<tr>
<td>( a_{2}^{SM,*} = 0.182 )</td>
<td>( a_2^{MP,*} = 0.0024 )</td>
</tr>
<tr>
<td>( U_{SM,*} = 0.182 )</td>
<td>( U^{MP,*} = 0.082 )</td>
</tr>
</tbody>
</table>

In the example equilibrium incentives in both periods under (SM) and full commitment (FC) are moderate (0.182) due to high positive autocorrelation \( \lambda \). If contracts must be renegotiation-
proof (RP) induced second-period effort increases to 0.48 under (SM). From an ex ante perspective second-period incentives of 0.48 are far too high and impose too much risk on the agent. To compensate for this effect the principal reduces first-period incentives from 0.182 to 0.018. Nonetheless, total equilibrium surplus for the principal in (SM) decreases from 0.182 under full commitment to 0.070 under renegotiation-proofness. Under market price measurement induced effort incentives under full commitment are lower than their (SM) counterparts due to the (unavoidable) use of the macroeconomic shock measure in incentive compensation. Therefore, equilibrium surplus $U^{\text{MP}*} = 0.082$ is significantly lower than under separate measurement. Similar to separate measurement, with market price contracting sequentially optimal second-period effective incentives are higher (0.0065) than under full commitment (0.0024). However, under (MP) incentives are closer to the full commitment optimum (0.182) than under separate measurement (0.48). This is the positive effect of market price measurement in an agency with limited commitment. In the example this effect is strong enough to outweigh the negative effect of using a non action-informative measure in both periods’ incentive contracts, $U^{\text{MP},R} = 0.081 > 0.070 = U^{\text{SM},R}$.

Proposition 1 shows that positively correlated earnings are key for market price measurement to be optimal. Christensen et al. (2005, p. 269) argue that while the analytical literature often assumes positively correlated earnings the empirical literature provides evidence for both positively and negatively correlated ones. An implication of our result for empirical research is that firms with positively correlated, informative earnings tend stronger to use market price measurement than firms with non- or negatively correlated earnings.

4.3 Market price measurement as a commitment to low incentives in an earnings management problem

In this section the agent’s earnings management action (only) introduces dynamics in the model. Consequently we now eliminate autocorrelated noise by setting $\lambda = \mu = 0$. With $\delta > 0$ and $g_1 \neq g_2$ earnings are given by:

$$y_{11} = g_1a_1 + \delta c + \varepsilon_{11}$$
$$y_{21} = g_2a_2 - \delta c + \varepsilon_{21}.$$

Market prices now reduce to

$$P_1 = K_1 + \beta (y_{11} + y_{12})$$
$$P_2 = K_2 + \beta (y_{11} + y_{12}) + \beta_2 (y_{21} + y_{22}).$$
with \( \beta = \frac{\sigma}{\sigma} v_1 \) and \( \beta_2 = \frac{\sigma}{\sigma} v_2 \). The effective incentive rates under (MP) as defined in (3) simplify to

\[
\bar{s}_1 = \beta (s_1 + s_2) \\
\bar{s}_2 = \beta_2 s_2.
\]

The agent’s ability to conduct earnings management creates a congruity-problem for the principal. Earnings management produces no benefits for the principal but is costly for her for two related reasons: The principal has to compensate the agent for his disutility from an unproductive effort. For this reason, optimal incentives will be less powered. This reduces not only incentives for earnings management but also for productive action in each period. As shown in appendix A2, (9) and (11), the incentives for earnings management are given by

\[
e = \delta (s_{11} - s_{21}) \text{ under SM (4)}
\]

\[
e = \delta (\bar{s}_1 - \bar{s}_2) \text{ under MP (5)}
\]

With \( g_1 = g_2 \) both periods would be identical. Hence, the incentive rates \( s_{11} \) and \( s_{21} \) under (SM) and \( \bar{s}_1 \) and \( \bar{s}_2 \) under (MP) would be identical at the optimum, too. Earnings management would not occur in equilibrium. With \( g_1 \neq g_2 \), however, as assumed, the agent has an incentive to move earnings to the period in which the incentive coefficient is higher.

Let \( G = (g_1 - g_2) \left(r\sigma^2 - g_1 g_2\right) \) in what follows. Before we analyze the limited commitment setting we present a full commitment result that helps to derive our main result.

**Lemma 5** Under full commitment and separate measurement: \( s_{11}^* > 0, s_{21}^* > 0 \). \( s_{11}^* \gtrless s_{21}^* \) iff \( G \gtrless 0, e^{SM,*} \gtrless \lesssim 0 \) iff \( G \lesssim \gtrless 0 \).

**Proof.** The results for \( s_{11}^* \) and \( s_{21}^* \) follow from table 2. \( e^{SM,*} \) is derived by inserting \( s_{11}^* \) and \( s_{21}^* \) from table 2 into (4).

Both periods’ earnings coefficients are positive to motivate positive effort. Efforts in period 1 and 2 affect earnings differently only due to different marginal contributions \( g_1 \) and \( g_2 \). However, a higher earnings sensitivity of first period effort compared to second period effort does not necessarily lead to higher incentives \( s_{11}^* > s_{21}^* \) in the first period. It does so, if and only if, the periods’ risk measured by \( r\sigma^2 \) is sufficiently high. In fact a period’s incentive rate \( s_{11} \) is only increasing in its effort sensitivity \( g_1 \) if the period’s risk \( r\sigma^2 \) is sufficiently high. To see this, notice

\[11\text{Recall that from assuming } \mu = \lambda = 0 \text{ it follows } \beta_{11} = \beta_{12} = \beta_1 \text{ and } \beta_{21} = \beta_{22} = \beta_2.\]

\[12\text{See Feltham and Xie (1994) for a detailed analysis of the congruity problem.}\]
that the first-best effort is \( a_t^{FB} = 1 \). The agent selects his second-best effort as \( a_t = s_{t1} g_t \) under (SM). If risk is low, it is optimal to induce second-best efforts \( a_t \) close to the first-best value 1. This will be reached by a \( s_{t1} \) close to \( 1/g_t \) which is decreasing in \( g_t \). Only if \( r \sigma^2 \) is sufficiently high \( s_{t1} \) becomes increasing in \( g_t \).

Thus, whether \( s_{11}^* - s_{21}^* \) is positive or negative in equilibrium does not depend on the impact of earnings management \( (\delta) \) at all as earnings management is identically costly in both periods. Rather, it depends on \( g_1, g_2, \) and \( r \sigma^2 \). Specifically, the sign of \( s_{11}^* - s_{21}^* \) is equal to the sign of the earnings sensitivity difference \( (g_1 - g_2) \) times the sign of the "risk to total sensitivity difference" \( r \sigma^2 - g_1 g_2 \): \( G = (g_1 - g_2)(r \sigma^2 - g_1 g_2) \). In what follows we therefore refer to \( G \) as the "sensitivity-to-risk" measure. As the agent selects \( e^{SM} = \delta(s_{11} - s_{21}) \) the sign of earnings management equals the sign of the sensitivity-to-risk measure \( G \).

With regard to the sign of earnings management a similar result as under (SM) applies under (MP) with the following difference: Under market price measurement the macroeconomic measure is included which increases the compensation risk of a period from \( r \sigma^2 \) to \( 2r \sigma^2 \). This – ceteris paribus – leads to lower incentive rates being optimal under (MP) compared to (SM). In particular, this holds for the second-period sequentially optimal incentive rate as the driving force in the renegotiation-proof setting:

**Lemma 6** *Sequentially optimal incentive weights under (SM) and (MP) are given by*

\[
\begin{align*}
  s_{21}^R &= \frac{g_2}{g_2^2 + r \sigma^2}, \\
  \bar{s}_2^R &= \frac{g_2}{g_2^2 + 2r \sigma^2}, \\
  s_{21}^R &> \frac{s_{21}^R}{s_{21}^R}.
\end{align*}
\]

**Proof.** See appendix B. 

Sequentially optimal contracting behavior ignores the effect of first-period earnings management \( e \) on second-period earnings. Rather, \( s_{21}^R \) and \( \bar{s}_2^R \) are solely determined to trade-off incentives for \( a_2 \) and second-period risk-sharing. Ignoring earnings management incentives ex post, however, causes agency costs ex ante. As under market pricing the macroeconomic shock will be contracted upon, again effective second-period incentives are lower than under separate measurement. For separate measurement we obtain:

**Lemma 7** \( s_{21}^* \overset{>}{\geq} s_{21}^R \) and \( e^{SM,*} \overset{<}{\leq} e^{SM,R} \) iff \( G \overset{>}{=} 0 \).

**Proof.** See appendix B.
Assume first $G > 0$. According to lemma 7 the second-period renegotiation-proof incentives under (SM) are too low compared to ex ante efficient ones. This induces extensive earnings management \( (e^{SM,R} > e^{SM,*} > 0) \). To compensate for that effect the principal reduces the first-period incentive rate \( s_{11}^R \) as well, but underproportionally in order to provide sufficient working incentives in period one. Under (MP) induced second-period incentives are even lower, \( s_{21}^R < s_{21}^* \) as is shown in lemma 6. Thus, for $G > 0$ separate measurement clearly dominates market price measurement under limited commitment.

For $G < 0$, however, there are potential benefits from implementing market price measurement. In this case second-period incentives under limited commitment in (SM) are too strong \( s_{21}^R > s_{21}^* \). Thus, using market price measurement potentially relaxes the renegotiation-proof constraint relative to (SM) as (MP)-measurement includes a commitment to lower second-period incentives.

**Proposition 2**  
a) Under limited commitment, if market price measurement dominates separate measurement in the earnings management problem, then \( g_1 > g_2 \) and \( r\sigma^2 < g_1g_2 \). b) Given the condition in a) applies, market price measurement dominates separate measurement if \( \delta \) and \( g_1 \) are sufficiently large.

**Proof.** See appendix B. ■

Lemma 7 restricts cases where market price measurement may dominate separate measurement to settings with \( G = (g_1 - g_2) (r\sigma^2 - g_1g_2) < 0 \). If \( G < 0 \) induced earnings management under separate measurement is negative. Under limited commitment second-period incentives are too high. In turn induced negative earnings management is even stronger, \( 0 > e^{SM,*} > e^{SM,R} \). Using market price measurement, the principal commits to lower second-period incentives which damps up excessive (negative) earnings management to some extent. Generally \( G < 0 \) occurs if either \( g_1 > g_2 \) and \( r\sigma^2 < g_1g_2 \) or \( g_1 < g_2 \) and \( r\sigma^2 > g_1g_2 \). Proposition 2, however, restricts the set of settings in which market pricing may dominate to the first scenario. In fact, the second scenario may as well result in a possible relaxation of sequentially optimal behavior, but the effect will always be overcompensated. To see this, notice that the higher the risk of the agency the lower the principal’s payoff. The lower the payoff, the lower ceteris paribus the benefits from relaxing sequentially optimal behavior. Moreover, a high second-period earnings sensitivity relative to the first period makes the second-period action more important. This implies that the cost of an exaggerated second-period incentive rate (that is set to exclusively control the second-period action) is comparatively low.

Part b) of the proposition provides a sufficient condition. Given the condition in a) applies, market pricing is the optimal measurement system if the earnings sensitivity of the first-period
action \((g_1)\) and the incentives for earnings management \((\delta)\) are sufficiently high. This coincides with intuition: If incentives for (negative) earnings management are strong, high renegotiation-proof second-period incentives under (SM) induce severe deviations from the ex ante optimal earnings management. In addition, with sufficiently high earnings sensitivity \(g_1\) low first-period incentives are optimal such that excessive (negative) earnings management is not counteracted by appropriately increasing the first-period incentive rate.

Again we conclude with a numerical example.

**Example 2:** Parameters of the example: \(g_1 = 5, g_2 = 1, r = 0.01, \sigma = 5,\) and \(\delta = 2.13\)

<table>
<thead>
<tr>
<th>Separate Measurement</th>
<th>Market Price Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC (s^*_1 = 0.22)</td>
<td>FC (\bar{s}^*_1 = 0.215)</td>
</tr>
<tr>
<td>RP (s^*_2 = 0.358)</td>
<td>RP (\bar{s}^*_2 = 0.338)</td>
</tr>
<tr>
<td>(a^*_1 = 1.099)</td>
<td>(a^{MP,*}_1 = 1.077)</td>
</tr>
<tr>
<td>(a^*_2 = 0.358)</td>
<td>(a^{SM,R}_2 = 0.80)</td>
</tr>
<tr>
<td>(e^{SM,*} = -0.276)</td>
<td>(e^{SM,R} = -1.039)</td>
</tr>
<tr>
<td>(U^{SM,*} = 0.729)</td>
<td>(U^{SM,R} = 0.269)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the example \(G < 0\) holds with \(g_1 > g_2\) and \(r\sigma^2 < g_1g_2\). Consider separate measurement first. Observe that the second-period incentive rate under limited commitment is too high (0.8 versus 0.358). This induces excessive (negative) earnings management. To counteract these incentives the principal increases \(s_{11}\) from 0.2 under commitment to 0.28 under limited commitment. However, the increase in \(s_{11}\) under limited commitment is by far underproportional compared to the increase in \(s_{21}\). As a consequence the agent is induced to heavily postpone earnings to the second period (−1.039 versus −0.276). The principal’s surplus declines from 0.729 to 0.269.

Under market price measurement we observe structurally identical effects. However, effective second-period incentive weight \(\bar{s}^*_2\) remains considerably below its (SM)-counterpart. It is closer to the full commitment optimum 0.358. This implies that earnings management incentives are ceteris paribus weaker. There is no need for a tremendous increase in first-period incentives to dam up earnings management. It follows that under limited commitment the principal’s surplus is higher under (MP) as compared to (SM).

\(^{13}\)Equilibrium values for variables presented in the table below do not depend on \(\sigma, v_1\) and \(v_2\).
5 Conclusion

In this paper we use a two-period agency model to compare earnings and market price based compensation. We assume that two performance measures, earnings and a macroeconomic measure, are publicly observed. Both measures are informative about terminal firm value but earnings only are informative about the agent’s effort. Accordingly, the firm’s stock price, or market price, aggregates both measures in order to provide a best estimate of terminal firm value. In an optimal long-term full commitment contract, however, the macroeconomic measure is never used, as it only adds noise to the manager’s compensation. Earnings turn out to be preferable as compared to market prices.

With this result in place we relax the full commitment assumption. We carry on assuming that the principal is free to offer a revised contract once she has observed first-period performance measure values. The agent may accept or deny the new contract. We model this setting by restricting the contracting space to renegotiation-proof contracts. Assuming limited commitment combined with interdependent periods introduces dynamics into our problem. We consider two types of interdependencies in separate settings. In the first one we assume that performance measures are correlated over periods. In the second one we allow for an earnings management activity decided upon in the first period, that reverses in the second. In both settings we are able to show that limited commitment creates agency costs. In both settings market price contracting is preferable to earnings contracting at least in some cases.

If earnings are positively correlated this creates additional risk and in turn reduces optimal incentive rates in a full commitment setting. With limited commitment, however, the second period incentive rate is chosen ex post optimally, ignoring the correlation. An excessive second period incentive rate results. Using a market price measure instead, similar effects arise. However, providing incentives is generally more costly and optimal incentive rates are lower. With limited commitment this results in lower deviation from the full commitment benchmark and may overcompensate the cost of using a non-informative measure.

If earnings management is possible, providing incentives for productive action at the same time creates incentives for earnings management. It is in that sense that incentive contracting becomes more costly and optimal incentive rates are lower. With limited commitment, second period incentive rates are set without considering first period earnings management choices. Again, if this results in excessive second period incentives with earnings, market price based compensation may become efficient.
Appendix

Appendix A: Full commitment solutions

1) Optimization problems in the autocorrelated noise setting \((\delta = 0, g_1 = g_2 = 1)\):

a) (SM), given that \(s_{12} = s_{22} = 0\) from lemma 1:

\[
\begin{align*}
\max & \quad a_1 + a_2 - f - s_{11} g_1 a_1 - s_{21} g_2 a_2 \\
\text{s.t.} & \quad CE^{SM} = f + s_{11} a_1 + s_{21} a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{r}{2} \sigma^2 (s_{11}^2 + s_{21}^2 + 2s_{11}s_{21}\lambda) \geq 0 \\
\quad & \quad a_1 = s_{11} \\
\quad & \quad a_2 = s_{21}
\end{align*}
\]

Inserting the binding participation constraint into the principal’s objective function we obtain

\[
a_1 + a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{r}{2} \sigma^2 [s_{11}^2 + s_{21}^2 + 2s_{11}s_{21}\lambda]. \tag{6}
\]

The objective function has to be maximized for \(s_{11}\) and \(s_{21}\) subject to the incentive constraints given above. The optimal solutions are

\[
\begin{align*}
s_{11}^* & = \frac{g_1 (r\sigma^2 + g_2^2) - g_2 \lambda r\sigma^2}{g_2^2 g_1^2 + (g_1^2 + g_2^2) r\sigma^2 + r^2 \sigma^4 (1 - \lambda^2)}, \\
s_{21}^* & = \frac{g_2 (r\sigma^2 + g_1^2) - g_1 \lambda r\sigma^2}{g_2^2 g_1^2 + (g_1^2 + g_2^2) r\sigma^2 + r^2 \sigma^4 (1 - \lambda^2)},
\end{align*}
\]

and the corresponding equilibrium surplus of the principal is given by

\[
U^{SM,*} = \frac{1}{2} \frac{2g_1^2 g_2^2 + \rho \lambda^2 (g_1^2 + g_2^2) - 2g_1 g_2 \lambda \rho \sigma^2}{2 g_2^2 g_1^2 + (g_1^2 + g_2^2) \rho \sigma^2 + r^2 \sigma^4 (1 - \lambda^2)}.
\]

b) (MP), we define \(\bar{s}_1 = s_1\beta + s_2\beta_{11}\) and \(\bar{s}_2 = \beta_{21}s_2\).

\[
\begin{align*}
\max & \quad a_1 + a_2 - E(f + s_1 P_1 + s_2 P_2) \\
\text{s.t.} & \quad CE^{MP} = E(f + s_1 P_1 + s_2 P_2) - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{r}{2} \sigma^2 V \geq 0 \\
\quad & \quad a_1 = \bar{s}_1 \\
\quad & \quad a_2 = \bar{s}_2
\end{align*}
\]

with \(V = [\bar{s}_1^2 + \bar{s}_2^2 + 2\bar{s}_1\bar{s}_2\lambda + (s_1\beta + s_2\beta_{12})^2 + s_2^2\beta_{22}^2 + 2(s_1\beta + s_2\beta_{12})s_2\beta_{22}\mu].\)
Inserting the binding participation constraint, the principal’s objective function under (MP) is given by
\[
\max a_1 + a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{r}{2} \sigma^2 \left[ \bar{s}_1^2 + \bar{s}_2^2 + 2s_1 s_2 \lambda + (s_1 \beta + s_2 \beta_{12})^2 + s_2^2 \beta_{22}^2 + 2(s_1 \beta + s_2 \beta_{12}) s_2 \beta_{22} \mu \right].
\]

Maximizing the objective function subject to \(a_1 = \bar{s}_1\) and \(a_2 = \bar{s}_2\) for \(s_1\) and \(s_2\) we obtain
\[
s_1^* = \frac{r \sigma^2 (\beta_{22} \mu (\beta_{11} + \beta_{21} - 2 \beta_{12}) - \beta_{12} - \beta_{22}^2 + \beta_{11} (\beta_{12} + \beta_{21} (1 - \lambda)) + \beta_{21} (\lambda - 1)) - \beta_{21}^2 + \beta_{11} \beta_{21}}{n},
\]
\[
s_2^* = \frac{r \sigma^2 (\beta_{21} \lambda - \beta_{11} + \beta_{12} + \beta_{22} \mu - 2 \beta_{21}) - \beta_{21}}{n}
\]
and
\[
\bar{s}_1 = s_1^* \beta + s_2^* \beta_{11},
\]
\[
\bar{s}_2 = \beta_{21} s_2^*
\]
with
\[
n = \beta [-2r^2 \sigma^4 \{ (\beta_{11} - \beta_{12}) (\lambda \beta_{21} - \mu \beta_{22}) + \beta_{21}^2 + \beta_{22}^2 - \beta_{11} \beta_{12} - \beta_{21} \beta_{22} \lambda \mu \}
+ 2r \sigma (\beta_{11} \beta_{12} + \beta_{22} \mu (\beta_{11} - \beta_{12})) - r \sigma^2 (\beta_{11}^2 + \beta_{12}^2 + 3 \beta_{21}^2 + \beta_{22}^2)
+ r^2 \sigma^4 (\beta_{12}^2 (\lambda^2 - 1) + \beta_{22}^2 \mu^2 - \beta_{11}^2) - \beta_{21}^2].
\]

The corresponding equilibrium surplus for the principal equals
\[
U_{MP,*} = \bar{s}_1^* + \bar{s}_2^* - \frac{(\bar{s}_1^*)^2}{2} - \frac{(\bar{s}_2^*)^2}{2} - \frac{r}{2} \sigma^2 \left[ \bar{s}_1^2 + \bar{s}_2^2 + 2\bar{s}_1 \bar{s}_2 \lambda + (s_1^* \beta + s_2^* \beta_{12})^2 + s_2^2 \beta_{22}^2 + 2(s_1^* \beta + s_2^* \beta_{12}) s_2 \beta_{22} \mu \right].
\]

2) Optimization problems in the earnings management setting \((\lambda = \mu = 0)\):

a) (SM):
\[
\max a_1 + a_2 - f - s_{11} (g_1 a_1 + \delta e) - s_{21} (g_2 a_2 - \delta e)
\]
\[
\text{s.t.}
\]
\[
CE^{SM} = f + s_{11} (g_1 a_1 + \delta e) + s_{21} (g_2 a_2 - \delta e) - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{e^2}{2} - \frac{r}{2} \sigma^2 (s_{11}^2 + s_{21}^2) \geq 0
\]
\[
a_1 = g_1 s_{11}
\]
\[
a_2 = g_2 s_{21}
\]
\[
e = (s_{11} - s_{21}) \delta.
\]
Taking the binding participation constraint into account, the problem becomes

$$\begin{align*}
\max & \quad a_1 + a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{\epsilon^2}{2} - \frac{r}{2} \sigma^2 (s_{11}^2 + s_{21}^2) \\
\text{s.t.} & \quad a_1 = g_1 s_{11}, a_2 = g_2 s_{21}, e = (s_{11} - s_{21})\delta.
\end{align*}$$

The optimal incentive rates and the expected surplus for the principal result as follows:

$$\begin{align*}
s_{11}^* &= \frac{\delta^2 g_2 + g_1 \left( g_2^2 + \delta^2 + r\sigma^2 \right)}{g_2^2 g_1^2 + (g_1^2 + g_2^2) \left( \delta^2 + r\sigma^2 \right) + 2\delta r \sigma^2 + r^2 \sigma^4}, \\
\frac{1}{2} g_2^2 g_1^2 + (g_1^2 + g_2^2) \left( \delta^2 + r\sigma^2 \right) + 2\delta r \sigma^2 + r^2 \sigma^4. \\
U_{SM,*} &= \frac{1}{2} \frac{\delta^2 + r\sigma^2 \left( g_1^2 + g_2^2 \right) + 2 \left( g_1 g_2 \delta + g_1^2 g_2^2 \right)}{g_2^2 g_1^2 + (g_1^2 + g_2^2) \left( \delta^2 + r\sigma^2 \right) + 2\delta r \sigma^2 + r^2 \sigma^4}.
\end{align*}$$

b) (MP):

$$\begin{align*}
\max & \quad a_1 + a_2 - E(f + s_1 P_1 + s_2 P_2) \\
\text{s.t.} & \quad C E^{MP} = E(f + s_1 P_1 + s_2 P_2) - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{\epsilon^2}{2} - \frac{r}{2} \sigma^2 [s_1^2 + \bar{s}_1^2 + (s_1 - s_2)^2 + s_2^2 + (s_2 - s_1)^2 + s_2^2 \beta_{s2}^2] \geq 0 \\
a_1 &= g_1 \bar{s}_1 \\
a_2 &= g_2 \bar{s}_2 \\
e &= \delta (\bar{s}_1 - \bar{s}_2).
\end{align*}$$

Inserting for the binding participation constraint the problem reduces to

$$\begin{align*}
\max & \quad a_1 + a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{\epsilon^2}{2} - \frac{r}{2} \sigma^2 [s_1^2 + \bar{s}_2^2 + (s_1 - s_2)^2 + s_2^2 \beta_{s2}^2] \\
\text{s.t.} & \quad a_1 = g_1 \bar{s}_1, a_2 = g_2 \bar{s}_2, e = \delta (\bar{s}_1 - \bar{s}_2).
\end{align*}$$

We get the following optimal solutions (we present only the effective incentive rates):

$$\begin{align*}
\bar{s}_1^* &= \frac{g_1 \left( g_2^2 + \delta^2 + 2r\sigma^2 \right) + g_2 \delta^2}{\delta^2 + 2r\sigma^2 + g_2^2 + 2r\sigma^2 (\delta^2 + r\sigma^2)}, \\
\bar{s}_2^* &= \frac{g_2 \left( g_1^2 + \delta^2 + 2r\sigma^2 \right) + g_1 \delta^2}{\delta^2 + 2r\sigma^2 + g_1^2 + 2r\sigma^2 (\delta^2 + r\sigma^2)}, \\
U_{MP,*} &= \frac{1}{2} \frac{\delta^2 + 2r\sigma^2 \left( g_1^2 + g_2^2 \right) + 2 \left( \delta^2 g_1 g_2 + g_1^2 g_2^2 \right)}{\delta^2 + 2r\sigma^2 + g_1^2 + g_2^2 + 4r\sigma^2 (\delta^2 + r\sigma^2)}.\end{align*}$$
Appendix B Proofs

Proof of Lemma 2

Consider the optimal incentive rates \( s_{11}^* \) and \( s_{21}^* \) under (SM). Both of them are strictly positive. Now assume that both or at least one effective incentive rate under (MP) are unequal to zero. Then by definition the macroeconomic measure will be element of the optimal managerial incentive contract under (MP). As all contracting variables can be contracted upon ex ante optimal including the macro measure in the compensation contract under (MP) is inefficient compared to separate measurement. Hence, the equilibrium payoff under (SM) must strictly exceed the equilibrium payoff under (MP). If \( s_{11}^* = s_{21}^* = 0 \) would be optimal under (MP) no effort could be induced which leads to an equilibrium payoff of zero for the principal. As under (SM) both incentive rates are strictly positive and \( s_{11}^* = s_{21}^* = 0 \) would lead to an equilibrium payoff of 0 under (SM) the principal’s surplus under (SM) is strictly higher than zero. Therefore, the principal’s equilibrium surplus under (SM) must exceed her equilibrium payoff under (MP) given full commitment is possible.

Proof of Lemma 3

a) At the beginning of the second period, the first-period action \( a_1 \) is already performed and the realizations of \( y_{11} \) and \( y_{12} \) have been observed. The principal’s ex post problem to determine the sequentially optimal second-period incentive weights under (SM) is to maximize the second-period part of its expected gross output, \( a_2 \), net of its cost subject to the incentive constraint for second-period effort. The cost covers the agent’s disutility of performing effort in period 2, \( a_2^2/2 \), and the risk premium \( \frac{r}{2} \text{Var}(s_{21}y_{21} + s_{22}y_{22}|y_{11}, y_{12}) \) to be paid to the agent:

\[
\max_{s_{21}, s_{22}} a_2 - \frac{a_2^2}{2} - \frac{r}{2} \text{Var}(s_{21}y_{21} + s_{22}y_{22}|y_{11}, y_{12})
\]

subject to \( a_2 = s_{21} \).

According to lemma 1 \( s_{22} = 0 \) at the optimum. The posterior variance \( \text{Var}(S_2|y_{11}, y_{12}) \) can be calculated as (with \( S_2 = s_{21}y_{21} \))

\[
\text{Var}(S_2|y_{11}, y_{12}) = \text{Var}(S_2) - \Sigma_{21} \Sigma_{y_{11}}^{-1} \Sigma_{21}^t
\]

with \( \Sigma_{21} = \begin{pmatrix} \text{Cov}(S_2, y_{11}) & \text{Cov}(S_2, y_{12}) \end{pmatrix} = \begin{pmatrix} s_{21}\sigma^2 \lambda & 0 \end{pmatrix} \). Hence,

\[
\text{Var}(S_2|y_{11}, y_{12}) = \sigma^2 s_{21}^2 (1 - \lambda^2).
\]

Inserting the incentive constraint into the objective function the principal’s problem reduces to

\[
\max U_2^{SM} = s_{21} - \frac{s_{21}^2}{2} - \frac{r}{2} \sigma^2 s_{21}^2 (1 - \lambda^2).
\]
From the first-order condition
\[
\frac{\partial U^*_{2}}{\partial s_{21}} = 1 - s_{21} - r\sigma^2 s_{21} (1 - \lambda^2) = 0
\]
one obtains the following sequentially optimal incentive weight on earnings
\[
s_{21}^R = \frac{1}{1 + r\sigma^2 (1 - \lambda^2)}.
\]

b) Similar to a) the principal’s problem under (MP) is given by
\[
\max_{s_{2}} a_{2} - \frac{a_{2}^2}{2} - \frac{r}{2} Var(s_{2}P_{2}|y_{11}, y_{12})
\]
subject to \(a_{2} = s_{2}\beta_{21}\).

The posterior variance can be calculated as
\[
Var(P_{2}|y_{11}, y_{12}) = Var(P_{2}) - \Sigma_{P_{2}|y_{1}}\Sigma_{y_{1}}^{-1}\Sigma_{P_{2}|y_{1}}'.
\]

With \(Var(P_{2}) = \sigma^2 [\beta_{11}^2 + \beta_{12}^2 + \beta_{21}^2 + \beta_{22}^2 + 2\beta_{11}\beta_{21}\lambda + 2\beta_{12}\beta_{22}\mu]\) and
\[
\Sigma_{P_{2}|y_{1}} = \begin{pmatrix}
\sigma^2 (\beta_{11} + \beta_{21}\lambda) & \sigma^2 (\beta_{12} + \beta_{22}\mu)
\end{pmatrix}
\]
one obtains
\[
Var(P_{2}|y_{11}, y_{12}) = \sigma^2 [\beta_{21}^2 (1 - \lambda^2) + \beta_{22}^2 (1 - \mu^2)].
\]

Inserting the variance term and the incentive constraint into (12) the principal’s problem reduces to
\[
U^*_2 MP = s_{2}\beta_{21} - \frac{(s_{2}\beta_{21})^2}{2} - \frac{r}{2}\sigma^2 s_{2}^2 [\beta_{21}^2 (1 - \lambda^2) + \beta_{22}^2 (1 - \mu^2)].
\]

From the first-order-condition
\[
\frac{dU^*_2}{ds_{2}} = \beta_{21} - s_{2}\beta_{21} - rs_{2}\sigma^2 [\beta_{21}^2 (1 - \lambda^2) + \beta_{22}^2 (1 - \mu^2)] = 0,
\]
we obtain the following sequentially optimal second-period incentive weight under (MP):
\[
s_{2}^R = \frac{\beta_{21}}{\beta_{21}^2 (1 + r\sigma^2 (1 - \lambda^2)) + \beta_{22}^2 r\sigma^2 (1 - \mu^2)}.
\]

With \(\pi_{2} = \beta_{21}s_{2}\) we obtain the effective incentive weight
\[
\pi_{2}^R = \frac{\beta_{21}^2}{\beta_{21}^2 (1 + r\sigma^2 (1 - \lambda^2)) + \beta_{22}^2 r\sigma^2 (1 - \mu^2)}.
\]

Proof of Lemma 4

a) The optimal (SM)-incentive weights under full commitment in the correlated noise setting are given in (7) in appendix A.
Calculating the differences of second-period incentive weights for (SM) under full commitment and renegotiation-proofness (as given by lemma 3) we obtain:

\[
s^*_2 - s^R_2 = \frac{-\lambda (\lambda + 1) r\sigma^2}{[1 + (1 - \lambda^2) r\sigma^2] [1 + (1 + \lambda) r\sigma^2]}.
\]  (13)

The denominator of (13) is strictly positive. The numerator is positive (zero, negative) iff \(\lambda\) is negative (zero, positive). Hence, \(s^*_2 - s^R_2 \geq 0\) iff \(\lambda \leq 0\). As \(a^{SM,(-)}_2 = s^{(\cdot)}_2\) the same relation applies for \(a_2\).

b) Taking the difference of renegotiation-proof second-period incentive rates \(\pi^R_2\) and \(s^R_2\) we obtain

\[
\pi^R_2 - s^R_2 = -\frac{r\sigma^2 p^2}{[1 + (1 - \lambda^2) r\sigma^2] [1 + (1 + \lambda) r\sigma^2]} (1 - \mu) (\mu + 1)
\]

\[
\beta^2_2 (1 + r\sigma^2 (1 - \lambda^2)) + \beta^2_2 r\sigma^2 (1 - \mu) \beta < 0.
\]

**Proof of Lemma 6**

a) Sequentially optimal incentives under (SM): The proof is analogue to the proof of lemma 3, part a), with \(\mu = \lambda = 0\) and with incentive compatibility constraint \(a_2 = g_2 s_2\). The principal’s objective function becomes

\[
\max U^{SM}_2 = s_2 g_2 - \frac{s_2^2 g_2^2}{2} - \frac{r\sigma^2 s_2}{2}.
\]

From the first-order-condition

\[
\frac{dU^{SM}_2}{ds_2} = g_2 - s_2 g_2^2 - r\sigma^2 s_2 = 0
\]

it follows \(s^R_2 = \frac{g_2}{g_2^2 + r\sigma^2}\).

b) Sequentially optimal incentives under (MP). The proof is analogue to part b) of the proof of lemma 3 with \(\mu = \lambda = 0\), \(\beta_2 = \beta_2 = \beta_2\) and with incentive compatibility constraint \(a_2 = g_2 s_2 \beta_2\). The principal’s objective function becomes

\[
U^{MP}_2 = g_2 s_2 \beta_2 - \frac{(g_2 s_2 \beta_2)^2}{2} - r\sigma^2 s_2 \beta_2^2.
\]

From the first-order-condition

\[
\frac{dU^{MP}_2}{ds_2} = g_2 \beta_2 - g_2^2 \beta_2^2 s_2 - 2r\sigma^2 \beta_2 s_2 = 0
\]

it follows \(s_2^R = \frac{g_2}{\beta_2 [g_2^2 + 2r\sigma^2]}\) and for the effective incentive rate \(\pi^R_2 = \beta_2 s_2 = \frac{g_2}{g_2^2 + 2r\sigma^2}\).

**Proof of Lemma 7**

\(s^*_2\) is given in (10) and \(s^R_2\) in proof of lemma 6. For the difference \(s^*_2 - s^R_2\) we obtain

\[
s^*_2 - s^R_2 = \frac{\delta^2 G}{\delta^2 G + (g_2^2 + g_2^2) (\delta^2 + r\sigma^2) + 2\delta_2 r\sigma^2 + r^2 \sigma^4} (g_2^2 + r\sigma^2) \leq 0\) if \(G \leq 0\).
From the proof of lemma 5 we know \( e^{SM,*} = \delta (s_{11} - s_{21}^*) = \frac{\delta G}{g_2^2 g_1^2 + (g_1^2 + g_2^2)(\delta^2 + r^2 \sigma^2) + 2\delta r \sigma^2 + r^2 \sigma^4} \). 
\( e^{SM,R} = \delta (s_{11}^R - s_{21}^R) \) can be calculated as
\[
e^{SM,R} = \frac{\delta G}{(g_2^2 + r \sigma^2)(g_1^2 + \delta^2 + r \sigma^2)}.
\]
with \( s_{11}^R \) given in (14). For the difference \( e^{SM,*} - e^{SM,R} \) we obtain
\[
e^{SM,*} - e^{SM,R} = -\frac{\delta^3 G (g_1^2 + r \sigma^2)}{[g_2^2 g_1^2 + (g_1^2 + g_2^2)(\delta^2 + r \sigma^2) + 2\delta r \sigma^2 + r^2 \sigma^4] (g_2^2 + \delta^2 + r \sigma^2)}(g_2^2 + \delta^2 + r \sigma^2)^2 \leq 0 \text{ iff } G \geq 0.
\]

**Proof of Proposition 2**

a) For \( G = 0 \) according to lemma 7 \( s_{21}^R = s_{21}^* \). Hence, the full commitment solution is induced under (SM) with renegotiation-proofness. As the full commitment payoff under (SM) exceeds always the full commitment payoff under (MP) (lemma 2), MP cannot be optimal for \( G = 0 \) under renegotiation-proofness. Assume \( G = (g_1 - g_2) (r \sigma^2 - g_1 g_2) > 0 \), then according to lemma 7 \( s_{21}^* > s_{21}^R \). Denote the principal’s maximum payoff as a function of \( s_{21} \) under SM as \( U^{SM} (s_{21}) \). This payoff function can be determined by solving the SM-problem in A2, a), for \( s_{11} \) as a function of \( s_{21} \) and inserting the solution into the principal’s objective function. \( U^{SM} (s_{21}) \) has the following properties: It is a strictly concave function which has a unique maximizer \( s_{21}^* \) and it is symmetric to its maximum. From lemma 6 we know that \( s_{21}^R < s_{21}^* \). Hence, even if market price contracting were "costless" and would only influence second-period incentives it were dominated as \( U^{SM} (s_{21}^R) > U^{SM} (s_{21}^*) \). As market price measurement is costly in terms of contracting on non-informative measures it directly follows that if market price contracting dominates separate measurement then \( G = (g_1 - g_2) (r \sigma^2 - g_1 g_2) < 0 \).

The difference of payoffs under limited commitment \( \Delta U^R = U^{SM,R} - U^{MP,R} \) can be written - see appendix C2 for details, as
\[
\Delta U^R = \frac{\sigma^2 g_1^2 (K^{pos} + K^0)}{2 (g_1^2 + \delta^2 + \sigma^2) (\mu^2 g_1^2 + \sigma^2)^2 (g_1^2 + \delta^2 + 2 \sigma^2) (\mu^2 g_1^2 + 2 \sigma^2)^2},
\]
where we have without loss of generality set \( g_2 = \vartheta g_1, \vartheta > 0 \) and \( r = 1, r_{14} \) with
\[
K^{pos} = 4 \sigma^8 + 2 \mu^6 g_1^6 \delta^2 + 2 \mu^7 g_1^6 \delta^2 + 2 \delta^4 \vartheta^2 + 2 g_1^4 \delta^2 \sigma^2 + 22 g_1^2 \delta^2 \sigma^2 \delta^4 + 6 g_1^4 \sigma^2 \delta^2 \delta^4 + 4 g_1^2 \delta^4 \delta^2 \vartheta + 6 g_1^2 \sigma^2 \delta^4 \vartheta + 4 \sigma^4 \delta^2 \vartheta + 12 \sigma^6 \delta^2 \vartheta + 2 g_1^4 \delta^2 \sigma^2 + 22 g_1^2 \delta^2 \sigma^2 + 22 g_1^4 \delta^2 \sigma^2 + 4 \sigma^8 \delta^2 \vartheta + 4 \vartheta^2 + \delta^2 \vartheta + 6 g_1^2 \vartheta^2 + 2 g_1^2 \vartheta^2 + 6 g_1^2 \sigma^2 \delta^4 + g_1^6 (\vartheta^2 + \vartheta^6) + 4 \sigma^8 \vartheta^2 > 0
\]
\(^{14} r = 1 \) is without loss of generality as \( r \) always appears in connection with \( \sigma, r \sigma^2 \). I.e. every change in \( r \sigma^2 \) can be captured by variation of \( \sigma \).
and

\[ K^\vartheta = (2g_1^4\delta^4\vartheta^4 + 2g_1^6\delta^2\vartheta^4) (\vartheta - 1) + [3\delta^2 g_1^4 \sigma^2 \vartheta^4 + 3\delta^4 g_1^2 \sigma^2 \vartheta^2 + 5\delta^2 g_1^2 \sigma^4 \vartheta^2] (\vartheta^2 - 1). \]

Therefore, for \( \Delta U^R < 0 \) it must hold that \( K^\vartheta < 0 \) which requires \( \vartheta < 1 \) or, \( g_1 > g_2 \), respectively. \( g_1 > g_2 \) in connection with \( G < 0 \) requires \( r\sigma^2 < g_1g_2 \). Hence, \( g_1 > g_2 \) and \( r\sigma^2 < g_1g_2 \) are necessary conditions for the optimality of market price contracting.

b) Assume \( g_1 > g_2 \) and \( r\sigma^2 < g_1g_2 \). Calculating the limits

i) \( \lim_{\delta \to \infty} \Delta U^R = -\frac{r\sigma^2 g_2(-2g_2^5 - 3g_2^3r\sigma^2 - 2g_1g_2^4 - 6g_1g_2^2r\sigma^2 - 4g_1r^2\sigma^4 + 2g_1^2g_2^2 + 3g_2^4r\sigma^2)}{2(g_2^2 + r\sigma^2)^2(g_2^2 + 2r\sigma^2)^2} \)

ii) \( \lim_{g_1 \to \infty} \Delta U^R = \frac{r\sigma^2 g_2(-2\delta^2 g_2^5 - 3\delta^2 r\sigma^2 + g_1^2 + 3g_2^2r\sigma^2 + 2r^2\sigma^4)}{22(g_2^2 + r\sigma^2)^2(g_2^2 + 2r\sigma^2)^2} \)

shows that i) becomes negative if \( g_1 \) is sufficiently high and ii) becomes negative if \( \delta \) is sufficiently high.

Appendix C  Payoffs and first-period incentive rates of renegotiation-proof problems

1) Autocorrelated noise setting

a) SM: We have to solve the full commitment objective function (6) from Appendix A1, a) subject to the (IC)-conditions \( a_1 = s_{11} \) and \( a_2 = s_{21} \) and subject to the renegotiation-proof constraint \( s_{21} = s_{21}^R = \frac{1}{1 + r\sigma^2(1 - \lambda^2)} \). The respective unconstrained objective function becomes

\[ \max_{s_{11}} s_{11} + s_{21}^R = \frac{s_{21}^R}{2} + \frac{r^2}{2} \sigma^2 \left( s_{11}^2 + (s_{21}^R)^2 + 2s_{11}s_{21}^R \lambda \right). \]

This optimization problem has the optimal solution

\[ s_{11}^R = \frac{r\sigma^2 \left( \lambda^2 + \lambda - 1 \right) - 1}{(r\sigma^2 \left( \lambda^2 - 1 \right) - 1) (1 + r\sigma^2)} \]

and the corresponding objective function value

\[ U^{SM,R} = \frac{r^2\sigma^4 \left( 2 - 3\lambda^2 + \lambda^4 - 2\lambda + 2\lambda^3 \right) + 2r\sigma^2 \left( 2 - 2\lambda^2 - \lambda \right) + 2}{2 \left( -1 - r\sigma^2(1 - \lambda^2) \right)^2 (1 + r\sigma^2)}. \]

b) MP: We have to solve the full commitment objective function (8) from Appendix A1, b) subject to the (IC)-conditions \( a_1 = \bar{s}_1 \) and \( a_2 = \bar{s}_2 \) and subject to the renegotiation-proof constraint
The solution to the problem is obtained if we solve the full commitment problem specified in \( a \), respectively. The respective unconstrained objective function becomes

\[
\max_{s_1, s_2} \bar{s}_1 + s_2 - \frac{\bar{s}_1^2}{2} - \frac{r}{2} \sigma^2 \left[ \bar{s}_1^2 + \left( s_2^R \right)^2 + 2 \bar{s}_1 s_2^R \lambda + \left( s_1^R \beta + s_2^R \beta_{12} \right)^2 + \left( s_2^R \right)^2 \beta_{22}^2 + 2 \left( s_1^R \beta + s_2^R \beta_{12} \right) s_2^R \beta_{22} \mu \right].
\]

This problem has the following solution

\[
s_1^R = \frac{\beta_{21}^2 r \sigma^2 (\lambda^2 - 1) - \beta_{21}^2 + \beta_{22} r \sigma^2 (\mu^2 - 1) + \beta_{21} \beta_{11} (1 + r \sigma^2) + r \sigma^2 \beta_{21} (\beta_{22} \mu + \beta_{21} \lambda + \beta_{21} \beta_{12})}{\beta (\beta_{21}^2 r \sigma^2 (\lambda^2 - 1) - \beta_{21}^2 + \beta_{22} r \sigma^2 (\mu^2 - 1)) (1 + 2 r \sigma^2)}.
\]

\[
s_2^R = \frac{\beta_{21}^2 r \sigma^2 (\lambda^2 - 1) - \beta_{21}^2 + \beta_{22} r \sigma^2 (\mu^2 - 1) + \beta_{21} \beta_{11} r \sigma^2 + r \sigma^2 \beta_{21} (\beta_{22} \mu + \beta_{21} \lambda + \beta_{21} \beta_{12})}{(\beta_{21}^2 r \sigma^2 (\lambda^2 - 1) - \beta_{21}^2 + \beta_{22} r \sigma^2 (\mu^2 - 1)) (1 + 2 r \sigma^2)}.
\]

The corresponding objective function value is given by

\[
U^{MP,R} = \bar{s}_1^R + s_2^R - \frac{\left( s_1^R \right)^2}{2} - \frac{\left( s_2^R \right)^2}{2} - \frac{r}{2} \sigma^2 \left[ \left( \bar{s}_1^R \right)^2 + \left( s_2^R \right)^2 + 2 \bar{s}_1 s_2^R \lambda + \left( s_1^R \beta + s_2^R \beta_{12} \right)^2 + \left( s_2^R \right)^2 \beta_{22}^2 + 2 \left( s_1^R \beta + s_2^R \beta_{12} \right) s_2^R \beta_{22} \mu \right].
\]

2) Earnings management setting

a) Solutions to the renegotiation-proof-problem under (SM)

The solution to the problem is obtained if we solve the full commitment problem specified in \( A2, a \), under the additional constraint \( s_{21} = s_{21}^R = \frac{g_2}{g_2^2 + r \sigma^2} \). This is the renegotiation-proofness condition derived in the proof of lemma 6:

\[
\max a_1 + a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{e^2}{2} - \frac{r}{2} \sigma^2 (s_{21}^R + s_{21}^R) \\
\text{subject to } a_1 = g_1 s_{11}, a_2 = g_2 s_{21}, e = \delta (s_{11} - s_{21})
\]

\[
s_{21} = \frac{g_2}{g_2^2 + r \sigma^2}.
\]

The optimal solution for \( s_{11} \) and the corresponding maximum surplus are given by

\[
s_{11}^R = \frac{\delta^2 g_2 + g_1 (g_2^2 + r \sigma^2)}{(g_2^2 + r \sigma^2) (g_1^2 + \delta^2 + r \sigma^2)}
\]

\[
U^{SM,R} = \frac{g_2 g_3 (3 r \sigma^2 - \delta^2 + 2 g_2^2) + r \sigma^2 (g_1^2 + g_2^2) + 2 g_1 g_2 \delta (g_2^2 + r \sigma^2) + g_4 (\delta^2 + r \sigma^2)}{(g_2^2 + r \sigma^2)^2 (g_1^2 + \delta^2 + r \sigma^2)}.
\]

b) Solutions to the renegotiation-proof-problem under (MP)

The solution to the problem is obtained if we solve the full commitment problem specified in \( A2, b \), under the additional constraint \( s_2 = s_2^R = \frac{g_2}{g_2^2 + 2 r \sigma^2} \) (see proof of lemma 6):

\[
\max a_1 + a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{e^2}{2} - \frac{r}{2} \left( \bar{s}_1 + \bar{s}_2 \right) + \left( s_1 \beta + s_2 \beta_{12} \right)^2 + \frac{g_2^2 \beta_{22}^2}{2}
\]

\[
\text{subject to } a_1 = g_1 \bar{s}_{11}, a_2 = g_2 \bar{s}_2, e = \delta (\bar{s}_{11} - \bar{s}_2)
\]

\[
s_2 = \frac{g_2}{g_2^2 + 2 r \sigma^2}.
\]
For the optimal incentive weight $s_1$ (and $\overline{s}_1$) and the principal’s maximum surplus we obtain

$$s^R_1 = \frac{g_2 (\beta_2 (\delta^2 + g_1 g_2) - \beta (g_1^2 + \delta^2 + 2r_\sigma^2)) + 2g_1 \beta_2 r \sigma^2}{\beta_2 (g_2^2 + 2r_\sigma^2) (g_1^2 + \delta^2 + 2r_\sigma^2)},$$

$$\overline{s}^R_1 = \frac{g_1 g_2^2 + 2g_1 r \sigma^2 + \delta^2 g_2}{(g_2^2 + 2r_\sigma^2) (g_1^2 + \delta^2 + 2r_\sigma^2)},$$

$$U^{MP,R} = \frac{1}{2} \frac{4r^2 \sigma^4 (g_1^2 + g_2^2) + 2r \sigma^2 (g_4^2 + 2\delta^2 g_1 g_2 + 3g_1^2 g_2^2) + g_2^4 (2g_1^2 + \delta^2) + g_2^2 \delta^2 g_1 (2g_2 - g_1)}{(g_2^2 + 2r_\sigma^2)^2 (g_1^2 + \delta^2 + 2r_\sigma^2)}. $$

References


